11 th
Polish
Combinatorial
Conference

ABSTRACTS

Będlewo, September 20–26, 2026

https://pcc.agh.edu.pl/

LIST OF TALKS

Sarosh Adenwalla

University of Liverpool

Boolean combinations of graph classes

Boolean combinations allow combining given combinatorial objects to obtain new, potentially more complicated, objects. In this paper, we initiate a systematic study of this idea applied to graphs. In order to understand expressive power and limitations of boolean combinations in this context, we investigate how they affect different combinatorial and structural properties of graphs, in particular χ-boundedness, as well as characterize the structure of boolean combinations of graphs from various classes.

This is joint work with Sam Braunfeld, John Sylvester, and Viktor Zamaraev.

Rui Alde Lopes
London School of Economics

New bounds for the torus variant of the no-three-in-a-line problem

The torus variant of the no-three-in-a-line problem seeks to determine the maximum amount of points that can be placed on a discrete $n \times m$ torus such that no three points lie in a line. We denote such a quantity by $\tau(n, m)$. In the torus, lines are the projection of lines in $\mathbb{Z}^2$, where each point (x, y) projects to $(x \mod n, y \mod m)$. The problem has been solved for tori $\mathbb{Z}_n \times \mathbb{Z}_m$ where m and n are coprime or $gcd(m, n) = p$ is a prime [**?**], however only trivial bound are known for square tori with composite dimensions.

In this work we give improved bounds for square tori $\mathbb{Z}_n^2$. We present a lower bound construction when $n = p^2$ for prime p, proving $\tau(p^2, p^2) \geq 2p$. We also show $p + 4 \leq \tau(2p, 2p) \leq \frac{88}{45}p + \frac{16}{9}$, improving the previous bounds of $p + 1 \leq \tau_{2p} \leq 2p + 2$ and solving a conjecture from [**?**]. Finally, we show that $\tau(n, n) = \Omega\left(\sqrt{\frac{n}{\log \log n}}\right)$ for every $n \geq 3$.

This is joint work with Brian Hearn and Cameron Strachan.

Recep Altar Ciceksiz
Umeå University

Induced poset saturation in the hypergrid

Given a poset P which can be embedded as an induced subposet of $[t]^n$, the induced poset saturation function $sat^*([t]^n, P)$ denotes the minimum size of a subset of $[t]^n$ that is both induced P-free and induced P-saturated. We show that for all $t \geq 2$, $sat^*([t]^n, P)$ satisfies a dichotomy: for every poset P, either there exists a constant C_P such that $sat^*([t]^n, P) = C_P$ for all n sufficiently large, or $sat^*([t]^n, P) = \Omega(\sqrt{n})$.

This is joint work with Victor Falgas-Ravry, Sabrina Lato, and Maryam Sharifzadeh.

Benjamin Bedert
University of Cambridge

The sum-free problem of Erdős

A set B is said to be sum-free if there are no x, y, z in B with $x + y = z$. One of the most elegant early applications of the probabilistic method in additive combinatorics is a classical argument of Erdős showing that any set of N integers contains a sum-free subset of size $N/3$, and this was later improved to $(N + 2)/3$ by Bourgain using an elaborate Fourier-analytic approach. A longstanding problem in additive combinatorics conjectures that one can always find a sum-free subset of size $N/3 + g(N)$, for some function g that tends to infinity. We discuss recent work confirming this conjecture.

This problem leads naturally to the study of estimating trigonometric sums. I will describe some new methods for doing this which combine Fourier analysis with ideas from additive combinatorics, and if time permits I will also discuss their applications to other problems, such as the lonely runner conjecture.

Paweł Rafał Bieliński
Warsaw University of Technology

MWIS in hereditary classes of ordered graphs

Maximum Weight Independent Sets is among the fundamental algorithmic problems in graph theory. We consider its complexity in the classes of ordered graphs that exclude a single ordered graph as an induced subgraph. In particular, we manage to extract significant algorithmic advantage by excluding any one of the three possible permutations of $2K_2$, and likewise for P_3 and K_2. On the other hand, we obtain NP-hardness results for any meaningfully larger forbidden induced subgraph. In the end, we formulate a near-complete complexity dichotomy.

Mindaugas Bloznelis & Jerzy Jaworski
Vilnius University & Adam Mickiewicz University

A peculiar measure concentration and its application to random mapping statistics

Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed non-negative integer valued random variables. Let $\mathcal{Y} = (Y_1, \ldots, Y_n)$ denote the random vector $(X_1, \ldots, X_n)$ conditioned on the event $X_1 + \cdots + X_n = n$. Its components $Y_1, \ldots, Y_n$ are exchangeable and they sum up to n. An instance of $\mathcal{Y}$ defines the class $M_{\mathcal{Y}}$ of mappings $\tau : [n] \to [n]$ with the property that the pre-images $\tau^{-1}(i) = \{j \in [n] : \tau(j) = i\}$ are of sizes y_i for $i \in [n]$. Note that each τ represents a functional oriented graph where each vertex j has one out-neighbor $\tau(i)$ and each vertex i has (possibly empty) set of in-neighbors $\tau^{-1}(i)$. In particular, y_i stands for the in-degree of the vertex i.

Interesting characteristics of a vertex i of mapping τ are the six-length $s(i)$, tail-length $t(i)$ and cycle-length $c(i)$ defined as follows. Start at vertex i and walk along the arcs $i \to \tau(i) \to \tau(\tau(i)) \ldots$. Such a walk eventually makes a cycle and stops. The number of visited vertices (including i) is the six-length $s(i)$. The number of cycle vertices is $c(i)$. The number of remaining vertices $t(i) = s(i) - c(i)$ counts the hop distance from i to the cycle.

We study asymptotic distributions as $n \to +\infty$ of the random variables $\mathbf{t}(\mathbf{i})$, $\mathbf{s}(\mathbf{i})$ and $\mathbf{c}(\mathbf{i})$, evaluated for τ selected uniformly at random from $M_{\mathcal{Y}}$ and for $\mathbf{i}$ selected uniformly at random from $[n]$. In our analysis we combine two steps: we establish a measure concentration of the distribution of $\mathcal{Y}$ and then apply results of Kevin Leckey and Nicholas Wormald (The Electronic Journal of Combinatorics, 2019) that describe the asymptotic distribution of $\mathbf{t}(\mathbf{i})$, $\mathbf{s}(\mathbf{i})$, $\mathbf{c}(\mathbf{i})$ in the case where τ is sampled from the class of mappings with a given (deterministic) sequence of in-degrees.

Peter Borg

University of Malta

Isolation of graphs

Given a set $\mathcal{F}$ of graphs, we call a copy of a graph in $\mathcal{F}$ an $\mathcal{F}$*-graph*. The $\mathcal{F}$*-isolation number* of a graph G, denoted by $\iota(G, \mathcal{F})$, is the size of a smallest subset D of the vertex set of G such that the closed neighbourhood $N[D]$ of D intersects the vertex sets of the $\mathcal{F}$-graphs contained by G (equivalently, $G - N[D]$ contains no $\mathcal{F}$-graph). If $\mathcal{F}$ consists of a 1-clique, then $\iota(G, \mathcal{F})$ is the *domination number* of G. If $\mathcal{F}$ consists of a 2-clique, then $\iota(G, \mathcal{F})$ is the *vertex-edge domination number* of G. The study of the general $\mathcal{F}$-isolation problem was introduced by Caro and Hansberg [6, 7]. This study is expanding very rapidly. A brief account of its development and of the speaker's recent work and collaborations in this field will be provided [1, 2, 3, 4, 5]. The speaker's collaboration with members of University of Gdańsk was funded by a SEA-EU grant awarded by the University of Malta.

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Bartłomiej Bychawski & Ivan Spyrydonov

Jagiellonian University

Quantitative bounds in the Polynomial Szemerédi Theorem for configuration $x, x+y, x+2y, x+y^3$

During the talk we will describe the proof of the quantitative version of the Polynomial Szemerédi Theorem for configuration

$$x, x+y, x+2y, x+y^3.$$

We proved, that every sufficiently large subset of integers A lying inside an interval $[1, N]$ contains desired configuration. More precisely, for some small positive constant $c > 0$ we obtained the bound

$$\#A \ll \frac{N}{\exp\big((\log\log\log N)^c\big)}$$

on the possible sizes of sets which lack the aforementioned pattern.

This particular configuration is interesting because it combines an arithmetic progression, which is tied by an algebraic relation

$$x + (x+2y) = 2(x+y),$$

with a polynomial y^3 of higher degree, which is algebraically independent but changes the relevant scale of possible y-s. Previous quantitative versions of the Polynomial Szemerédi Theorem only cover its subconfigurations and use slightly different arguments for each of them.

Previously to our work, Borys Kuca showed a quantitative bound for this configuration over finite fields, which are often the test case for results of this flavor. The main difficulty of our work was to handle the importance of scale, as if $y > N^{1/3}$ then $y^3 > N$, hence we need only to focus on values of y smaller than a cubic root of the length of our interval. This issue is not visible in the test case and it was not addressed previously.

This is joint work with Bartosz Głowacki.

Stijn Cambie

KU Leuven

Resolutions of #34 and #35 in the graph theory collection

In this talk, we give the main ingredients for the proof of two problems within Ramsey theory posed by Erdős, Faudree, Rousseau and Schelp, which are listed as problem 34 and 35 in the graph theory collection, and as problem 569 and 570 in the database erdosproblems.com.

For two graphs G and H, the Ramsey number $R(G, H)$ is defined as the smallest N such that in every red-blue colouring of the edges of the complete graph K_N, there is a red copy of G or a blue copy of H.

The result says that for every k and every graph H with m edges and no isolated vertices, $R(C_k, H) \leq (k-1)m + 1 \leq km$ and provided that m is sufficiently large with respect to k, $R(C_k, H) = 2m + \lfloor \frac{k-1}{2} \rfloor$. That is, we obtain both a general bound and exact asymptotic bound.

This is joint work with Andrea Freschi, Patryk Morawski, Kalina Petrova and Alexey Pokrovskiy.

Demetres Christofides

INSPIRE Research Centre

Diameter thresholds of random Cayley graphs

Given a finite group G, the random Cayley graph model $\mathcal{G}(G,p)$ is defined as the probability space of all Cayley graphs of G where each element of G is included in the generating set independently at random with probability p. While $\mathcal{G}(G,p)$ shares several macroscopic similarities with the standard Erdős-Rényi model $G(n,p)$, such as threshold phenomena, its inherently regular and algebraic structure yields important differences.

In this talk, we will explore the threshold probabilities for the diameter of random graphs drawn from $\mathcal{G}(G,p)$. For a fixed target diameter $d \geqslant 2$, it is well-known that in $G(n,p)$, the threshold for transitioning to diameter at most d is tightly concentrated around $p \sim \sqrt[d]{\frac{2\log n}{n^{d-1}}}$. We will demonstrate that random Cayley graphs exhibit a similar phenomenon up to a constant factor.

Specifically, we establish that for any $\varepsilon > 0$, a graph in $\mathcal{G}(G,p)$, where G is a group of order N, has diameter at most d with high probability if

$$p \geqslant \sqrt[d]{(1+\varepsilon)d!\frac{\log N}{N^{d-1}}}$$

and diameter greater than d if

$$p \leqslant \sqrt[d]{\frac{1-\varepsilon}{2^d} \cdot \frac{\log N}{N^{d-1}}}$$

Crucially, we will highlight how the exact threshold constants in $\mathcal{G}(G,p)$, unlike in $G(n,p)$, depend fundamentally on the underlying family of groups being considered. We will conclude by discussing specific group families that demonstrate both our upper and lower bounds are essentially best possible.

This is joint work with Klas Markström, and Christina Savvidou.

Maciej Cisiński
AGH University

Further progress on Wojda's conjecture

Two digraphs of order n are said to pack if they can be found as edge-disjoint subgraphs of the complete digraph of order n. It is well established that if the sum of the sizes of the two digraphs is at most $2n-2$, then they pack, with this bound being sharp. However, it is sufficient for the size of the smaller digraph to be only slightly below n for the sum of their sizes to significantly exceed this threshold while still guaranteeing the existence of a packing.

In 1985, Wojda conjectured that for any $2 \leq m \leq n/2$, if one digraph has size at most $n-m$ and the other has size less than $2n - \lfloor n/m \rfloor$, then the two digraphs pack. It was previously known that this conjecture holds for $m = \Omega(\sqrt{n})$. We confirm it for $m \geq 93$.

This is joint work with A. Żak.

Sebastian Czerwiński

University of Zielona Góra

Packing-list coloring of graphs

The pack-list coloring is a generalization of colorings from list assignments. Given an assignment to the k-list L, a L pack is a collection of k pairwise edge-disjoint colorings, each selecting one color from every list. This framework naturally extends classical list coloring by requiring several compatible list colorings to coexist simultaneously.

We study the *packing proper connection*, where every coloring in the packing is proper-connected, that is, between every pair of vertices there exists a properly edge-colored path. The minimum list size guaranteeing such a packing is called the proper packing connection number and is denoted by $pc_\ell^*(G)$.

We also introduce a stronger variant, the *one-path packing proper connection*, where for every pair of vertices, there exists a single path that is properly colored in every layer of the packing.

We also briefly discuss several other packing-list coloring variants.

This is joint work with Jarosław Grytczuk and Małgorzata Śleszyńska-Nowak and Michał Tuczyński.

Jadwiga Czyżewska

University of Warsaw

Induced Erdős-Pósa property for long holes, thetas, and beyond

We say that a family of graphs has the Erdős-Pósa property if for some graph H belonging to this family we have the following: every graph G and k, G either admits k disjoint copies of H, or we can delete a set X of cardinality $f(k)$ such that $G - X$ is H-minor-free. In our work, we study the induced Erdős-Pósa property, where we require the copies of H to be not only vertex-disjoint but also anti-adjacent. We also assume a more natural definition of a small hitting set: instead of hitting the objects with a set X, we do it with the neighborhood of a few vertices, namely $N[X]$.

We show that long induced cycles and long induced thetas admit the induced Erdős-Pósa property, which resolves a special case of a conjecture of Ahn, Gollin, Huynh, and Kwon.

We also prove that graphs excluding k long induced thetas as induced minors admit balanced separators consisting of the neighborhood of $\mathcal{O}(tk \log k)$ vertices. This, in turn, resolves a special case of a conjecture of Gartland and Lokshtanov and, combined with known techniques, yields a QPTAS for Maximum Weight Independent Set in this class of graphs, as well as for the problem of finding a largest induced subgraph of bounded treewidth satisfying a hereditary CMSO_2 formula.

This is joint work with Tomáš Masařík, Marcin Pilipczuk, Amadeus Reinald, and Paweł Rzążewski.

Magda Dettlaff

University of Gdańsk

Majority C-coloring of graphs

Inspired by the majority colorings and C-colorings, we introduce and study the majority C-coloring of graphs [C. Bujtás, M. Dettlaff, H. Furmańczyk, A. Laskowska, *Majority C-coloring of graphs*, 2026]. In such a vertex coloring, every vertex shares its color with at least half of its neighbors.

Formally, let $G = (V, E)$ be an undirected graph and k a positive integer. A *majority C-coloring* (C$_\geqslant$-coloring, for short) of G with k colors is a surjective function $\phi : V \to [k]$ such that every vertex $v \in V$ satisfies $\big|\{\, u \in N(v) : \phi(u) = \phi(v)\,\}\big| \geq \frac{1}{2}\deg_G(v)$. The largest integer k for which G admits a C$_\geqslant$-coloring is called *majority C-chromatic number* (C$_\geqslant$-chromatic number, for short) and it is denoted by $\overline{\chi}_\geqslant(G)$.

An upper bound on $\overline{\chi}_\geqslant(G)$ is proved in terms of the order, minimum, and maximum degree. Its sharpness is demonstrated by several results over different graph classes. In particular, $\overline{\chi}_\geqslant(P_n^k) = \overline{\chi}_\geqslant(C_n^k) = \lfloor n/(k+1) \rfloor$ is true for the k-th power of a path and a cycle if $n \geq k+2$. Further, $\overline{\chi}_\geqslant(G) = \frac{1}{3}(n-d)$ holds if G is a (claw, K_4)-free cubic graph and contains d diamonds. It is also shown that the majority C-chromatic number is not monotone under edge deletion. In fact, both the lower and upper bounds are sharp in the inequality chain $\overline{\chi}_\geqslant(G) - 2 \leq \overline{\chi}_\geqslant(G-e) \leq \overline{\chi}_\geqslant(G) + 1$. The minimum and maximum number of edges in an n-vertex graph G with $\overline{\chi}_\geqslant(G) = k$ are determined for every n and k. It is also pointed out that the classical chromatic number $\chi(G)$ and $\overline{\chi}_\geqslant(G)$ are incomparable, and the difference $\overline{\chi}_\geqslant(G) - \chi(G)$ can take any positive or negative integer. On the other hand, $\overline{\chi}_\geqslant(G) + \chi(G) \leq n+1$ holds for every graph G of order n. The decision problem of whether $\overline{\chi}_\geqslant(G) \geq k$ holds is NP-complete for every fixed $k \geq 2$. In contrast, some sufficient conditions for $\overline{\chi}_\geqslant(G) \geq 2$ are proved, and a linear-time algorithm is presented that determines $\overline{\chi}_\geqslant(T)$ if T is a tree.

This is joint work with Csilla Bujtás, Hanna Furmańczyk, and Aleksandra Laskowska.

Wouter van Doorn

Dense interval sums of unit fractions

Write the sum $\frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{b}$ as one fraction $\frac{u_{a,b}}{v_{a,b}}$, with $u_{a,b}$ and $v_{a,b}$ positive and coprime. For a fixed a, we provide bounds on the smallest $b = b(a)$ for which $v_{a,b} < v_{a,b-1}$, solving a problem of Erdős and Graham.

Cezary Dudkiewicz

Adam Mickiewicz University

Asymptotic normality of the giant component in random intersection graphs

In a random intersection graph $\mathbb{G}(n, m, p)$, there is a set of n vertices and m attributes, with any pair of vertex and attribute being connected with probability p. Edges between vertices are added if they share an attribute. We are currently studying the size of the giant component near the critical window of the phase transition in this model.

Previous results include the law of large numbers by M. Behrisch (2007) as well as asymptotic normality by L. Dong and Z. Hu (2022) for cases where the average degree is constant. In the sparse supercritical regime where it slowly tends to 1, LLN was proven by B. Wang, L. Wang and K. Xiang (2018). Our research focuses on proving asymptotic normality for this range by using an exploration process and the martingale central limit theorem. This talk will cover our result, as well as remaining problems.

This is joint work with K. Rybarczyk.

Kunal Dutta

University of Warsaw

Finding near-optimal centerpoints via Tverberg points

An α-centerpoint of a set of n input points in $\mathbb{R}^d$, is a point such that any halfspace containing it, also contains an α-fraction of the input points. Helly's theorem implies the existence of a $1/(d+1)$-centerpoint for every finite set of points in $\mathbb{R}^d$. The question of *finding* such a centerpoint has algorithmic significance and a long history. Since the seminal work of Clarkson, Eppstein, Miller, Sturtivant and Teng [IJCGA, 1996], the fastest algorithmic options in this direction, have been either to find $\Omega(1/d^2)$-centerpoints in polynomial time in d [Clarkson et al. IJCGA, 1996, Har-Peled and Jones, SoCG 2019] or to find $\Omega(1/d)$-centerpoints, in time $O(d^d)$. Recently a remarkable result of Cherapanamjeri [*FOCS*, 2024] gave the first polynomial time randomized algorithm for computing *approximate* $\Omega(1/d)$-centerpoints, that is a point ε-close to the halfspaces that define any $\Omega(1/d)$-centerpoint. However the algorithm takes significantly more time than $O(d^{16})$, as it involves running an interior-point based subroutine for $\Omega\left(\left(\frac{d}{\varepsilon}\right)^{16}\right)$ steps.

We give a $\tilde{O}(d^5)$ time randomized algorithm for computing *exact* $\Omega\left((d \cdot \ln d \cdot \ln \ln d)^{-1}\right)$-centerpoints. Our output quality is within a $\frac{1}{\ln d \ln \ln d}$-factor of $(d+1)^{-1}$, which is the best guarantee possible. This improves significantly on the longstanding $d^{O(d)}$ running time of Clarkson et al. for obtaining $\omega(1/d^2)$-centerpoints.

Our main tool is a new randomized algorithm to compute points in the intersection of Tverberg partitions using a lift of Sarkaria [*Isr. J. Math.* 1991], together with sampling ideas from Soberón [*Combinatorica*, 2019].

As a simple corollary, we also use these ideas to obtain a new polynomial-time (in d) algorithm for the *Colorful Carathéodory* problem, which improves upon the $d^{O(\log d)}$ runtime algorithm of Mulzer and Stein [SoCG, 2015].

If time permits, I shall also discuss some further developments including an approximation algorithm for the Colorful Carathéodory problem, and its applications to the problem of finding Tverberg centers and eventually (approximate) $\Omega(1/d)$ centerpoints.

Anna Flaszczyńska

AGH University

Local irregularity of k-multigraphs

A locally irregular graph is a graph in which every pair of adjacent vertices has different degrees. A locally irregular coloring of a graph G is an edge coloring in which each color class induces a locally irregular subgraph.

The definition of locally irregular multigraphs is analogous to the definition of locally irregular graphs. An edge coloring of G is called locally irregular if each color class induces a locally irregular submultigraph. The locally irregular chromatic index $\mathrm{lir}(G)$ is the smallest number of colors needed for a locally irregular coloring of G. A k-multigraph is a multigraph obtained from a graph G by replacing each edge of G with k parallel edges and is denoted by ${}^kG$.

In [2], Grzelec and Woźniak proposed the Local Irregularity Conjecture for 2-multigraphs, which was generalized for other k-multigraphs in the following way in [1].

Conjecture (Local Irregularity Conjecture for k-multigraphs [1]) For every connected graph $G \neq K_2$ and every integer $k > 1$, the k-multigraph ${}^kG$, obtained from G by replacing each edge with k parallel edges, satisfies $\mathrm{lir}({}^kG) \leq 2$.

This is joint work with Igor Grzelec, and Alfréd Onderko.

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Wojciech Gabryelski

Wrocław University of Science and Technology

Boltzmann sampling and optimal exact-size sampling for directed acyclic graphs

We propose two efficient algorithms for generating uniform random directed acyclic graphs with a randomly varying size, as well as an asymptotically optimal exact-size sampler based on them, that performs $\frac{n^2}{2} + o(n^2)$ operations and requests to a random generator. This was achieved by extending the Boltzmann model for graphical generating functions and by using various decompositions of directed acyclic graphs. The presented samplers improve upon the state-of-the-art algorithms in terms of theoretical complexity, offer a significant speed-up in practice, and open perspectives regarding edge-constrained sampling.

This is joint work with Zbigniew Gołębiewski, and Martin Pèpin.

Krystian Gajdzica
Jagiellonian University

On the partition function of a rectangle

A partition of a natural number n is a finite non-increasing sequence of positive integers whose sum is n. The total number of such sequences for n is counted by the partition function $p(n)$.

The story of integer partitions goes back to Euler, who, among other things, proved the celebrated pentagonal number theorem, and discovered the generating function for the partition function $p(n)$:

$$\sum_{n=0}^{\infty} p(n)q^n = \prod_{n=1}^{\infty} \frac{1}{1-q^n}.$$

There is an abundance of ways to generalize the partition function. We introduce its "geometric" extension, and investigate the number $p(m,n)$ of ways to partition a rectangle of size $m \times n$ into rectangular blocks with integer sides, where two partitions of the rectangle are considered the same if they consist of the same multiset of blocks (their geometric arrangement is neglected). We prove some basic properties of $p(m,n)$ and exhibit an analogue of the Hardy-Ramanujan formula in the case of $p(2,n)$. Moreover, we also describe the asymptotic behavior of the number of restricted partitions of a rectangle, where only blocks of some special sizes can be used as parts.

This is joint work with Robin Visser, and Maciej Zakarczemny.

Frederik Garbe

Czech Academy of Sciences

Universally spectral and trace positive matrix words

We discuss the following question: which words in noncommuting matrix variables and their formal transposes universally evaluate to matrices with nonnegative eigenvalues or nonnegative trace, regardless of the dimension or entries of the substituted matrices?

In this monomial setting, we provide a complete structural characterization analogous to Hermitian-square representations. This contrasts sharply with the broader noncommutative polynomial setting, where trace positivity is known not to be captured by Hermitian-square-type certificates. For individual words, however, we prove that both spectral and trace positivity force a rigid cyclic combinatorial square structure. This rigidity also extends naturally to the complex and infinite-dimensional settings.

As an application of our main theorems, we resolve questions originally posed by Lieb and Pierce, and later conjectured by Hillar and Johnson, motivated by quantum statistical mechanics. Our approach establishes a surprising connection to graph theory and yields a matrix analogue of the positive graph conjecture of Camarena, Csóka, Hubai, Lippner, and Lovász.

This is joint work with Fan Wei.

Aleksandra Gorzkowska

AGH University

Independent domination in planar graphs

A set S of vertices in a graph G is a dominating set of G if every vertex not in S is adjacent to a vertex in S. An independent dominating set in G is a dominating set of G with the additional property that it is an independent set. The independent domination number, $i(G)$, is the minimum cardinality among all independent dominating sets in G. It is known that for any planar graph G of order n with minimum degree at least 2, it holds that $i(G) \leq \frac{1}{2}n$ [1]. In the talk we present an infinite family of planar graphs G with minimum degree 2 that achieve equality in the bound. It is known that for any tree T of order n, it holds that $i(T) \leq \frac{1}{2}n$. We also provide a constructive characterization of all trees T that achieve equality in the bound.

This is joint work with Michael A. Henning, Elżbieta Kleszcz, and Monika Pilśniak.

References

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Mariusz Grech
Wrocław University of Science and Technology

Wreath products and automorphism groups of edge-colored graphs

We study the converse of the classical results of Sabidussi and Hemminger on automorphism groups of graph compositions. Given a graph (or colored graph, digraph) whose automorphism group is a wreath product $A \wr B$, we ask whether it necessarily arises from the corresponding graph construction and whether A and B are themselves automorphism groups of the factors. For the natural imprimitive action, we obtain mainly positive results. We also investigate the product action of wreath products, where the situation is more intricate and leads to new open problems related to hypergraphs.

Hubert Grochowski

Warsaw University of Technology

Computational and probabilistic aspects of the shortest path under attacks game

Real life situations in which the data can be imprecise or uncertain can be modeled using two-player games. In this talk, we present a min-max two-player game called *Shortest Path under Attacks*, played on a weighted graph, between two players: the Pather, whose target is to choose a path between two fixed vertices, and the Attacker, whose target is to distribute a given budget over the edges and consequently, increase the weights of some of them.

During the talk, we will focus on the mixed-strategy variant of the game, where each player wants to find an optimal probability distribution over their possible actions. More precisely, we will prove that, in certain special cases, optimal strategies for the players can be found in polynomial time. Moreover, we will demonstrate that, in these cases, an optimal mixed strategy for the Pather is given by a probability distribution derived from the uniform distribution.

This is joint work with Armin Fügenschuh, and Konstanty Junosza-Szaniawski.

Igor Grzelec
AGH University

Locally irregular total coloring and strong locally irregular total coloring of graphs

We say that a graph is *locally irregular* if adjacent vertices have different degrees. After a short introduction about the Local Irregularity Conjecture [1] and improved version of this conjecture stated by Sedlar and Škrekovski in [3] we present the concept of strongly locally irregular graphs introduced by Bensmail and Marcille in [2]. Then we introduce the Local Irregularity Conjecture for total graphs, where *total graph* is an ordered triple (V_0, V_1, E), where V_0, V_1 are the sets of empty and full vertices, respectively, $V_0 \cap V_1 = \emptyset$, and the set of edges E is a subset of $\binom{V_0 \cup V_1}{2}$ $(E \cap (V_0 \cup V_1) = \emptyset)$. By a *locally irregular* total graph $G = (V_0, V_1, E)$ we mean the total graph G where every pair of adjacent vertices has different *total degrees*. The *total degree* of a vertex v is the number of edges in E containing v plus one if $v \in V_1$ or plus zero otherwise. Then we present the set of results supporting this conjecture. In the end, we present the strong variant of this problem and the initial results on it.

This is joint work with Anna Flaszczyńska, Aleksandra Gorzkowska, Alfréd Onderko, and Mariusz Woźniak.

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Andrzej Grzesik
Jagiellonian University

Making a K_4-free graph balanced bipartite

We show that every K_4-free graph on n vertices can be made balanced bipartite by removing at most $n^2/9$ edges. This proves the conjecture of Balogh, Clemen and Lidický [1], as well as generalizes Sudakov's result [3] on the bipartite distance of K_4-free graphs and Reiher's result [2] on the sparse half of K_4-free graphs.

This is joint work with József Balogh, Ignacy Buczek and Piotr Kuc.

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Grzegorz Gutowski

Jagiellonian University

Graph division games

Motivated by the controller placement problems in software-defined networks and the fair division principles of classical "cake cutting", we investigate the following two-player zero-sum game. In our model, a defender places a limited number of controllers on graph vertices, while an attacker deletes a limited number of vertices. The defender score is the total number of surviving vertices reachable from any remaining controller. We formalize the computational problems associated with various game dynamics (defender plays first; attacker plays first; players play simultaneously; pure or mixed strategies). We show that these natural problems are NP-complete or even Σ_2^{P}-complete, depending on the specific variant.

This is joint work with Konstanty Junosza-Szaniawski, Antonio Lauerbach, and Alexander Wolff.

Hamed Hatami

McGill University

A characterization of idempotent Schur multipliers

The celebrated Cohen-Host idempotent theorem characterizes idempotents in the Fourier algebra as finite signed sums of indicators of cosets of open subgroups. Quantitative refinements due to Green-Sanders in the abelian case, and Sanders in the non-abelian setting, use tools from additive combinatorics to bound the number of cosets in terms of the Fourier algebra norm.

I will discuss the analogous problem for Schur multipliers. We prove that every idempotent Schur multiplier is a finite signed sum of contractive ones, with at most $2^{O(\gamma^4)}$ summands when the norm is at most γ. This answers a question of Katavolos and Paulsen from 2003.

After our paper was made public, Carenini observed that the main structural lemma in our proof can be applied to recover the Cohen–Host theorem and its quantitative refinements by a considerably simpler argument that avoids additive combinatorics. In the non-abelian case, this also improves Sanders's triple-exponential bound to a single-exponential one.

This talk is based on joint work with Csongor Beke, Marcel Goh, Sean Jaffe, and Daniel Naylor.

Brian Hearn

London School of Economics

Spanning tight components in 4-uniform hypergraphs

A subgraph G of a hypergraph H is *spanning* if every vertex of H lies in an edge of G, and is *tightly connected* if there is a tight walk between any two edges of G. We prove that every n-vertex 4-uniform hypergraph with minimum codegree at least $\lfloor n/4 \rfloor$ has a spanning tight component. This is tight, and settles the 4-uniform case of a conjecture of Illingworth, Lang, Müyesser, Parczyk, and Sgueglia.

This is joint work with Francesco Di Braccio, Joanna Lada, Mihir Neve, and Lu-Ming Zhang.

Jędrzej Hodor

Jagiellonian University

Invitation to torso parameters of annotated graphs

An annotated graph is a pair (G, S) where G is a graph and S is a subset of vertices of G. The vertices in S are *important* vertices of the graph. We would like to decompose G so that the *complexity* of the vertices of S is preserved but we do not care much about the other vertices. Several groups recently discovered that it is useful in this context to consider so-called *torso decompositions*, and *torso parameters*. Torso parameters can be described in terms of rooted minors, i.e. minors witnessed by a model where each branchset contains a vertex in S. We obtain several excluded minor – bounded parameter dualities analogous to these in classical structural graph theory. I will discuss and carefully introduce the main concepts along with some applications and open problems.

This is based on joint work with Hoang La, Piotr Micek, Clément Rambaud; and Quentin Claus, Gwenaël Joret, Pat Morin; and Maël Dumas, Tomasz Mazur, Piotr Micek, Clément Rambaud, Giannos Stamoulis.

Nadzieja Hodur

AGH University

Distinguishing index of $K_{1,s}$-free graphs - new results and generalizations

The distinguishing index $D'(G)$ of a graph G is the smallest number k such that G has an edge-k-colouring breaking all nontrivial automorphisms of G. The parameter was first defined by Kalinowski and Pilśniak in 2015; they proved that in general, with exception of certain small cycles, there is $D'(G) \leq \Delta(G)$. For $K_{1,s}$-free graphs it is possible to obtain a better bound. In [1] there was proved that for connected, $K_{1,s}$-free graphs such that $|G| \geq 6$ the distinguishing index is bounded by $s-1$. In this talk, we present further improvement of this bound and other possible generalisations of the result.

This is joint work with Christoph Brause, and Aleksandra Gorzkowska.

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Filip Jasionowicz
University of Oxford

Finding induced detours in linear time on interval and circular-arc graphs

The problem of finding shortest paths in graphs is well explored. Meanwhile, finding detours, i.e. non-shortest paths, is an active area of research. We provide $O(n+m)$-time algorithms for finding induced non-shortest path on interval and circular-arc graphs with n vertices and m edges.

As to the state of the art, for interval graphs, a reduction to Ishizeki, Otachi, and Yamazaki's algorithm for finding longest induced paths in graphs of bounded chordality solves the problem in time $O(nm)$. For circular-arc graphs, the literature does not mention any algorithm improving on Chiu and Lu's $O(n^{4.75})$-time one for general graphs.

Moreover, we extend our techniques to the parametrised setting that asks for induced paths which are by at least k longer than a shortest one, and we show how to solve the problem on circular-arc graphs in time $O(n^2m)$.

Finally, we turn to finding detours in directed graphs. Here, we refute a win/win method that has been repeatedly suggested and unsuccessfully attempted in the literature.

This is joint work with Sandra Kiefer.

Rafał Kalinowski

AGH University

Strong majority colourings of graphs

Motivated by the notions of majority vertex-colourings of graphs and digraphs, as well as majority edge-colourings of graphs, we propose and study strong majority colourings.

A strong majority vertex-colouring as a function $c : V(G) \to C$ such that for any vertex $v \in V(G)$ and any colour $\alpha \in C$, at most half of the neighbours of v have colour α. The minimum number of colours in such a colouring we denote by $\mathrm{Maj}(G)$. It occurs that there exist graphs with arbitrarily large $\mathrm{Maj}(G)$. However, there is a tight upper bound $\mathrm{Maj}(G) \le 2\Delta(G) + 1$ for every graph with minimum degree $\delta(G) \ge 2$.

A strong majority edge-colouring is a function $c : E(G) \to C$ such that for every edge $e \in E(G)$ and every colour $\alpha \in C$, at most half of the edges adjacent to e have colour α. Let $\mathrm{Maj}'(G)$ denote the minimum number of colours in such an edge-colouring of G. There exist a constant upper bound for $\mathrm{Maj}'(G)$ for all finite graphs G. We conjecture that this constant is as small as 4. We confirm this conjecture for some classes of graphs.

This is joint work with Mateusz Kamyczura, Monika Pilśniak, and Mariusz Woźniak.

Mateusz Kamyczura

AGH University

Conflict-free chromatic index

Let G be a graph with maximum degree Δ. An edge coloring of G is called closed conflict-free if the closed neighborhood of every edge contains a uniquely colored element. The open variant is defined analogously, excluding the edge itself from its neighborhood. The minimum number of colors required for such colorings are denoted as the closed and open conflict-free chromatic indices of G, respectively.

While the closed variant was previously known to be bounded by $3\log_2\Delta + 4$, the best known general upper bound for the open setting was $O(\log^{2+\epsilon}\Delta)$. In this work, we prove that for both variants, the conflict-free chromatic index is bounded above by $(1 + o(1))\log_2\Delta$ for all graphs with sufficiently large Δ. Since complete graphs require at least $(1-o(1))\log_2\Delta$ colors in both settings, our general upper bound is asymptotically tight in order and in the leading constant. Our proof combines the probabilistic method, including the Lovász Local Lemma and Chernoff bounds, with deterministic graph decomposition techniques.

This is joint work with Jakub Przybyło.

Jasmin Katz

London School of Economics

Zero-sum Ramsey numbers of bounded degree graphs

Zero-sum Ramsey numbers were defined by Bialostocki and Dierker in 1990 as an algebraic variant of the well-known Ramsey number. Given a graph G and a finite Abelian group $(\Gamma, +)$ such that $|\Gamma|$ divides $e(G)$, the *zero-sum Ramsey number* of G in Γ, denoted by $R(G, \Gamma)$, is the minimum integer t such that for any edge-colouring $c : E(K_t) \to \Gamma$, there is a copy G' of G in K_t such that $\sum_{e \in E(G')} c(e) = 0_\Gamma$.

We prove that a linear upper bound of the form $R(G, \Gamma) \leq Cn$ holds for every bounded degree n-vertex graph G and finite Abelian group Γ, where $|\Gamma|$ divides $e(G)$. Here C depends only on the maximum degree of G. This bound is tight up to the value of the multiplicative constant.

This is joint work with Xiaopan Lian, Alexandru Malekshahian, and Andrey Shapiro.

Balázs Keszegh

Alfréd Rényi Institute of Mathematics & ELTE

Saturation of oriented graphs

We consider a natural extension of the classical saturation problem to oriented graphs. We study the minimum number of edges in an oriented graph H on n vertices that avoids a fixed oriented graph G, but adding any edge to H creates a copy of G.

Our main result shows that saturation functions of oriented graphs attain at least four distinct orders of magnitude. If G is acyclic, then its saturation function is bounded if and only if G contains an isolated edge; otherwise, it is linear. When the forbidden graph consists of a tournament together with an isolated edge, we identify nontrivial families whose saturation functions are bounded, as well as families whose saturation functions are linear. The remaining two orders of magnitude are $\Theta(n\sqrt{\log n})$ and $\Theta(n \log n)$. The corresponding lower bounds are obtained by adapting a recently developed technique for the saturation problem in edge-ordered graphs, while the matching upper bounds are constructive. The first family consists of a specific class of oriented graphs, while the second applies to a broader family of oriented graphs in which every edge is contained in an oriented triangle, subject to mild additional assumptions.

Finally, we also study the semisaturation problem, in which we require only that adding any missing edge to H creates a copy of G. For this variant, we completely characterize the possible orders of magnitude: the semisaturation function is either bounded, linear, or $\Theta(n \log n)$.

This is joint work with Vladimir Bošković.

Bartłomiej Kielak
Leipzig University

Solution of uniform Turán's Tetrahedron Problem

Abstract

Turán's Tetrahedron Problem asks to determine the Turán density of the complete hypergraph $K_4^{(3)}$ (tetrahedron). This problem, posed by Turán in 1941, is one of the most famous problems in extremal combinatorics and its solution would attract $500 prize from Erdős. In the 1980s, Erdős and Sós asked to determine Turán densities of $K_4^{(3)-}$ (broken tetrahedron) and $K_4^{(3)}$ (tetrahedron) when edges are constrained to be uniformly distributed in the host hypergraph. The presumably easier case of the broken tetrahedron was solved by Glebov, Král' and Volec [Israel J. Math. 211 (2016), 349–366] and Reiher, Rödl and Schacht [J. Eur. Math. Soc. 20 (2018), 1139–1159]. We solve the tetrahedron case by proving that the uniform Turán density of $K_4^{(3)}$ is equal to 1/2; this confirms that Rödl's lower bound construction from 1986 is optimal.

This is joint work with Daniel Král', Ander Lamaison, Hong Liu, Xichao Shu, and Zhuo Wu.

Tomasz Kościuszko

Adam Mickiewicz University

Extremal behaviour of linear equations

Let the first N positive integers $1, 2, 3, \ldots, N$ be coloured with a fixed number of colours. When are we guaranteed to find integers in the same colour that satisfy a given linear equation? In other words, what is the threshold value of N such that a colouring avoiding any (non-trivial) solution is no longer possible? This is a fundamental question studied as early as the beginning of the 20th century by Schur, Rado and later Roth, Szemerédi and many others. The answer and the tools used for proofs vary depending on the equation we choose. In this talk, we will describe a couple of our results in this area, making the important distinction between translation invariant equations and non-invariant equations. In the first category, we will present improved bounds for long invariant equations and the main ingredient: the new version of the Bogolyubov-Ruzsa lemma. In the second category, we will describe improved bounds for Schur-like numbers as well as a generalised version of Schur numbers for longer equations.

Krzysztof Kosiorowski

Jagiellonian University

Coarse Menger's Theorem for two paths

The "coarse Menger's conjecture" states that for any $k, d \geq 1$ there exists $\ell > 0$ such that for every graph G and any $S, T \subseteq V(G)$ either there exist k paths between S and T pairwise at distance at least d, or there exists $X \subseteq V(G)$ such that $|X| \leq k-1$ and every path between S and T is at distance at most ℓ from X. The conjecture in general is refuted, though it remains true or is still open under specific conditions. We focus on the case $k = 2$, which is already known to be true. We improve the best known bound on ℓ.

This is a joint work with Piotr Micek.

Julia Kozik

AGH University

On the integrity of cartesian products of graphs

Vertex integrity and edge integrity are classical graph vulnerability parameters. While they have been extensively studied for various graph classes, relatively little is known about their behavior on Cartesian products. Benko, Ernst, and Lanphier established asymptotic bounds on the vertex integrity of planar graphs in terms of graph order. Motivated by their work, this talk presents recent results on the vertex and edge integrity of Cartesian products of graphs, including grid graphs, products involving paths and cycles, and several other families of Cartesian products.

This is joint work with Andrzej Żak.

Tomasz Krawczyk

Warsaw University of Technology

Characterizing circular-arc graphs by minimal forbidden induced subgraphs

Circular-arc graphs are the intersection graphs of arcs on a circle. This graph class is hereditary, that is, it is closed under vertex deletion, and therefore it can be characterized by the family of minimal non-circular-arc graphs. In this talk, I will present the main ideas and techniques behind the characterization of this family.

Joint work with Yixin Cao.

Michael Krivelevich

Tel Aviv University

On the unreasonable effectiveness of vertex sampling in graph theoretic problems

Based on the research experience of the speaker, we will showcase several quite diverse graph theoretic problems, where vertex percolation or more generally vertex sampling play a crucial role, despite their apparent irrelevance to the problem at hand.

Adam Krupowies
Adam Mickiewicz University

Ramsey numbers of family $\mathcal{P}_3$

We study multicolor Ramsey numbers for the family $\mathcal{P}_3$ of 3-uniform minimal paths of length three. We prove that $R(r;\mathcal{P}_3) = r+5$ for every $1 \leq r \leq 11$. Our approach is based on Turán-type methods. As a key ingredient, we determine the relevant Turán numbers and higher-order Turán numbers for $\mathcal{P}_3$, together with the corresponding extremal 3-uniform hypergraphs. By analyzing them we find the upper bound for $R(r;\mathcal{P}_3)$. The lower bound is found by special construction.

Jakub Kwaśny
AGH University

On the sequence variant of the $1-2-3$ conjecture

Let G be a connected graph other than K_2, and let $\prec$ be a total order on $E(G)$. For an edge weighting $w : E(G) \to S$, we define a vertex colouring of G as follows: the colour $f(v)$ of a vertex v is the sequence $(w(e_1), \ldots, w(e_k))$ of weights of all edges incident to v, listed according to $\prec$. Seamone and Stevens studied whether, for a given graph and total order, there exists an edge weighting w which yields a proper vertex colouring in this way.

We investigate the inverse problem: given a graph G and an edge weighting w does there exist a total order on $E(G)$ that induces a proper vertex colouring? We provide partial results for the case when w is a proper edge colouring.

This is joint work with Aleksandra Gorzkowska.

Hoang La

LISN

Graph reconstruction via subgraph queries

Bastide, Cook, Erickson, Groenland, Kreveld, Mannens, and Vermeulen (WG 2023) introduced the problem of reconstructing graphs from queries on triples of vertices, where each query asks whether the induced subgraph is connected or disconnected. This framework was later generalized by Galliot, La, Maistre, Petiteau, and Watel (EUROCOMB 2025), who considered partitions of the set of all isomorphism classes of graphs: the answer to a query is the part containing the isomorphism class of the induced subgraph. For instance, connectedness queries correspond to the bipartition in which one part contains all connected graphs and the other all disconnected graphs, up to isomorphism. This generalized setting also captures the classical Graph Reconstruction Conjecture of Kelly (1957) and Ulam (1960), which asserts that every graph on at least three vertices is uniquely determined by its induced subgraphs obtained by deleting one vertex.

Kluk, La, and Piecyk (WG 2024) showed that reconstructing graphs from variants of connectedness queries is hard, and that even for restricted graph classes, graphs with identical answers to all queries need not be unique. Motivated by this, Galliot et al. asked which partitions, for general tuples of vertices, guarantee unique reconstruction for all sufficiently large graphs.

For every integer $k \geq 2$, we give necessary and sufficient conditions on a partition Π of the isomorphism classes of graphs of order k such that every graph on at least $R(R(k))$ vertices is uniquely reconstructible from its answers to all k-vertex queries with respect to Π, where R denotes the diagonal Ramsey number. Our proof is constructive and yields a reconstruction algorithm using all queries, with running time $O(n^{Ck})$ for some fixed constant C. We also prove a lower bound of order $R(k)$: under only the necessary conditions on Π, there exist many graphs on fewer than $R(k)$ vertices that are not uniquely determined by their query answers.

This is joint work with Johanne Cohen, and Bianca Oancea.

Andrew Lane
University of Passau

On the factorization of Dirac hypergraphs

A celebrated result from the 70s due to Baranyai states that the complete k-uniform hypergraph on n vertices decomposes into perfect matchings if $k \mid n$. We prove a minimum degree version of this result. Note that a necessary condition for a k-uniform hypergraph G on n vertices to admit a decomposition into perfect matchings is that G is vertex-regular and $k \mid n$. We show that, if this is satisfied and in addition G has minimum $(k-1)$-degree at least $(\frac{1}{2} + o(1))n$, then it indeed decomposes into perfect matchings. The constant $1/2$ is best possible, as below this threshold, G might not even contain a single perfect matching. This settles a conjecture of Glock, Kühn, and Osthus from 2021.

This is joint work with Yangyang Cheng and Stefan Glock.

Michał Lasoń

Polish Academy of Sciences

Arboricity nearly bounds degeneracy

Arboricity and degeneracy are two fundamental and closely related graph parameters that measure the sparsity of a graph. Every k-degenerate graph is k-arboric, but some k-arboric graphs are only $(2k-1)$-degenerate. These basic observations lead to a natural structural question:

How far are k-arboric graphs from being k-degenerate?

Specifically, what are the strongest constraints on the edges removed from a k-arboric graph in order to get a k-degenerate graph? The simplest instance:

Can every 2-arboric graph be made 2-degenerate after removing a matching?

More specifically, a (d,h)*-decomposition* of a graph G is a partition of its edge set E into two subsets E_1 and E_2 such that:

- edges E_1 span a d-degenerate graph,
- edges E_2 span a h-bounded-degree graph.

Our main result is a characterisation of all possible decomposition types (d,h) that exist for all k-arboric graphs. In particular, we determine whether every k-arboric graph admits a $(k,k-1)$-decomposition, that is, a decomposition into a k-degenerate graph and a $(k-1)$-bounded-degree graph.

This is joint work with Bartłomiej Bosek, Grzegorz Gutowski, and Jakub Przybyło. It is based on our paper.

Florian Lehner

University of Auckland

Groups acting on trees and tree-like graphs

Bass-Serre theory is the perhaps most important tool for studying subgroups of the automorphism group of a tree. However, its usefulness in constructing group actions with given local properties is limited. To overcome this issue, Reid and Smith recently introduced the theory of local action diagrams. These diagrams are purely combinatorial objects which are in 1-to-1 correspondence to (P)-closed groups acting on trees.

The aim of this talk is to introduce graphs of group actions, a notion generalising local action diagrams while retaining many interesting properties of the theory, in particular constructability of groups from local combinatorial information.

This is joint work with Christian Lindorfer, Rögnvaldur Möller, and Wolfgang Woess.

Bartłomiej Lewandowski

University of Warsaw

Coarse properties of vertex-minor closed classes of graphs

A graph H is a *vertex-minor* of G if it can be obtained from G via a sequence of local complementations and vertex deletions. Vertex-minors exhibit numerous structural and algorithmic properties analogous to those of traditional graph minors. The class of circle graphs is vertex-minor closed and serves as a fundamental building block in Geelen's Structure Conjecture, the vertex-minor analog to the Robertson–Seymour Graph Minors Structure Theorem. This conjecture describes graphs in a proper vertex-minor closed class in terms of circle graphs, subjected to two operations: bounded perturbations and a more intricate construction. Although Geelen's conjecture does not hold in full generality, these objects and operations remain of independent interest from the vertex-minor perspective.

Parallel to these developments, the emerging field of Coarse Graph Theory has begun bridging coarse geometry and structural graph theory, as explored in the works of Georgakopoulos and Papasoglu, the "Asymptotic Structure" series by Nguyen, Scott, and Seymour, and many other works. Central to this field is the study of *coarse tree decompositions*, i.e. tree decompositions whose bags are coverable by a fixed number of balls of bounded radius.

As part of an effort to bridge these two areas of research, we present bounds on coarse properties for graph classes related to Geelen's Structure Conjecture. In particular, we provide bounds on the coarse treewidth of circle graphs and of bounded perturbations of graphs in proper vertex-minor closed classes.

These are results of my Master's thesis project under supervision of Michał Pilipczuk.

Anna Margarethe Limbach

Charles University

On Khovanskii's Theorem and its application to the Erdős multiplication table problem

By Khovanskii's theorem, for a finite set of integer vectors A and a positive integer N, the set NA of N-fold sums of elements from A grows eventually polynomially in N, i.e., there is a polynomial p_A and an integer N_{Kh} such that $|NA| = p_A(N)$ for all $N \geq N_{Kh}$.

In 2023, Granville, Shakan and Walker published an effective version of Khovanskii's theorem, i.e., they gave an upper bound on N_{Kh}. This bound was consecutively improved by Granville, Smith and Walker. By using Graver bases, we find a more structural proof of an improved upper bound.

In 1955, Erdős posed the problem of determining the order of magnitude of $|P(2,n)|$ and proved that $|P(2,n)| = o(n^2)$ for $n \to \infty$, where $P(k,n)$ is the set of products with k factors from the set $\{1,\dots,n\}$. Darda and Hujdurović asked in 2015 whether, for each fixed n, $|P(k,n)|$ is a polynomial in k of degree $\pi(n)$ - the number of primes not larger than n.

While their question remains open, we can show that $|P(k,n)|$ grows eventually polynomially by encoding the problem as k-fold sums. For each integer n, there is a polynomial q_n of degree $\pi(n)$ as well as a non-negative integer k_n such that $|P(k,n)| = q_n(k)$ for each $k \geq k_n$. Moreover, we give estimates for k_n and for the leading coefficient of q_n.

This is joint work with Robert Scheidweiler, and Eberhard Triesch.

Dingyuan Liu

Karlsruhe Institute of Technology

Turán problems for simplicial complexes

An abstract simplicial complex $\mathbf{F}$ is a non-uniform hypergraph without isolated vertices whose edge set is closed under taking subsets. The extremal number $\mathrm{ex}(n, \mathbf{F})$ is defined as the maximum number of edges in an $\mathbf{F}$-free simplicial complex on n vertices. Although Turán-type problems for simplicial complexes have long appeared in extremal set theory, a systematic study of $\mathrm{ex}(n, \mathbf{F})$ was initiated only recently by Conlon, Piga, and Schülke. In contrast to uniform hypergraphs, even the order of magnitude of $\mathrm{ex}(n, \mathbf{F})$ remains unknown for most simplicial complexes.

In this talk, we present a general framework for estimating $\mathrm{ex}(n, \mathbf{F})$ via generalised Turán numbers of associated hypergraphs. Using this approach, we determine the asymptotic behaviour, and in some cases the exact value, of $\mathrm{ex}(n, \mathbf{F})$ for large classes of simplicial complexes, extending a result of Conlon, Piga, and Schülke. Several open problems concerning the potential behaviour of $\mathrm{ex}(n, \mathbf{F})$ are formulated.

This is joint work with Maria Axenovich, Dániel Gerbner, and Balázs Patkós.

Piotr Micek

Jagiellonian University

Asymptotic dimension of graphs excluding an induced or fat minor

Asymptotic dimension is a large-scale invariant of metric spaces that was introduced by Gromov (1993). A major open problem in coarse graph theory asks whether, for every fixed graph H, the class of graphs excluding H as a fat minor has bounded asymptotic dimension. A weaker conjecture asserts the same conclusion for graphs excluding H as an induced minor.

We substantially expand the range of graphs H for which these conjectures are known to hold. Specifically, we prove bounded asymptotic dimension for the following classes of graphs: graphs excluding a fixed planar graph as a fat minor; and graphs excluding a fixed string graph as an induced minor. All of our results also hold for the stronger notion of Assouad–Nagata dimension, a refinement of asymptotic dimension.

These results have a number of applications, two of which we mention here. First they imply that an induced minor-closed class is χ-bounded if and only if it does not contain some Burling graph. Second they imply that, in graph classes excluding a fixed planar graph as an induced minor, treewidth is polynomial in maximum degree.

This is joint work with Robert Hickingbotham, Freddie Illingworth, and Bartosz Walczak.

Wojciech Michalczuk

Wrocław University of Science and Technology

NORMAL APPROXIMATION FOR SUBGRAPH COUNT IN GRAPHON-BASED RANDOM GRAPHS

Given a symmetric, measurable function $W : [0,1]^2 \to [0,1]$, vertices in a W-*random graph* are assigned random latent positions $U_i \sim \mathcal{U}[0,1]$ and are connected with location-dependent probabilities $W(U_i, U_j)$. The function W is commonly referred to as a *graphon*. Graphon-based framework covers a wide class of random graphs, including random geometric graphs and stochastic block models.

The analysis of the *subgraph count* – the number N^G of copies of a fixed graph G within a random graph – provides major insight into the structure of random networks. Recently in [1], the asymptotic normality of N_n^G, as $n \to \infty$, has been characterized in the *sparse setting* (where $W_n = p_n W$ for some graphon W and edge density parameters $p_n \ll 1$). Moreover, bounds for the normal approximation in the Kolmogorov distance were established for strongly balanced graphs G, see [2].

In this talk, we present improved normal convergence rates for the subgraph count in the sparse regime with arbitrary graph G. Additionally, we address the normal approximation in the absence of the sparsity condition on W_n, focusing specifically on edge and triangle counts.

This is based on a joint work with Grzegorz Serafin.

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Piotr Miska

Jagiellonian University

Connections between certain numbers related to derangements and r-permutations

For non-negative integer parameters r, u, m, n define

$$
\begin{aligned}
\mathcal{D}(r,u,m,n) := \Big\{\ \sigma \in \mathcal{S}_{r+n} \ \Big|\ & \sigma(x) = y \text{ for exactly } u \text{ pairs } (x,y) \\
& \ \text{such that } 1 \le x, y \le r \text{ and} \\
& \sigma(t) = t \text{ for exactly } m \text{ elements } r+1 \le t \le r+n \ \Big\}, \\
\mathcal{D}_{r,u,m}(n) := \Big\{\ \sigma \in \mathcal{S}_{r+n} \ \Big|\ & \forall_{1 \le x < y \le r}\ x \text{ and } y \text{ are in disjoint cycles of } \sigma \text{ and} \\
& \sigma(z) = z \text{ for exactly } u \text{ elements } 1 \le z \le r, \text{ and} \\
& \sigma(t) = t \text{ for exactly } m \text{ elements } r+1 \le t \le r+n \ \Big\},
\end{aligned}
$$

where $\mathcal{S}_n$ denotes the set of all the permutations of the set $\{1, \ldots, n\}$.

The aim of the talk is to show connections between the sets $\mathcal{D}(r,u,m,n)$, $\mathcal{D}_{r,u,m}(n)$, and the sets of (some classes of) r-derangements.

This is joint work with Błażej Żmija.

Patryk Morawski
ETH Zurich

Almost tight bounds for induced subdivisions

Subdivisions of complete graphs play a central role in combinatorics, having deep connections to structural, extremal, and topological aspects of graph theory. A celebrated conjecture of Mader, proved independently by Bollobás and Thomason and by Komlós and Szemerédi, states that every graph of average degree of order h^2 contains a subdivision of K_h.

We consider the induced variant of this problem. A theorem of Kühn and Osthus implies that, for every fixed graph H and every $s \geq 1$, graphs of sufficiently large average degree contain either a copy of $K_{s,s}$ or an induced subdivision of H. However, even for $H = K_h$, the best previous quantitative bounds were far from optimal.

We prove nearly tight bounds for forcing induced subdivisions of K_h. We show that every $K_{s,t}$-free graph of average degree $\Omega_{s,t}(h^{2(s-1)} \log^{7(s-1)} h)$ contains an induced subdivision of K_h, and that every C_{2k}-free graph with $k \geq 3$ and average degree $\Omega_k(h \log^5 h)$ contains an induced subdivision of K_h. These bounds substantially improve the previously known results and are nearly optimal in both settings. They also hold if K_h is replaced by any other graph on h vertices.

This is joint work with Zach Hunter, Aleksa Milojević, and Benny Sudakov.

Anna Nenca
University of Gdańsk

Local computation of parity in cellular automata

Cellular automata (CAs) provide a simple yet powerful model of distributed computation in which each cell updates its state according to a fixed local rule. Despite relying exclusively on local interactions, cellular automata are capable of performing nontrivial global computations, making them an attractive framework for studying the relationship between locality and computational complexity.

Among the best-known benchmark problems in this area is the parity problem. Given a binary cyclic configuration of odd length, the objective is to design a cellular automaton that converges to the homogeneous configuration of all 0s when the initial number of 1s is even, and to the homogeneous configuration of all 1s otherwise. Since each cell has access only to a finite neighborhood, solving the problem requires the automaton to compute a global property without centralized control.

For many years it was unknown whether such a cellular automaton exists. The first proposed solution, known as the BFO rule, marked a significant breakthrough, although subsequent work revealed inaccuracies in its original formulation. A corrected version, accompanied by a complete proof of correctness, eventually established that the parity problem is indeed solvable.

The existence of a solution naturally raises a number of further questions. In particular, it remains unclear whether the parity problem can be solved by a simpler cellular automaton and what the minimum neighborhood size required for such a solution is. Recent advances provide new tools for investigating these questions.

This is joint work with Barbara Wolnik.

Karolina Okrasa

Warsaw University of Technology

Hypergraph colourings and strong sparsification

The classic notion of (vertex) graph colouring can be generalized to hypergraphs in several interesting ways; the most popular one avoids monochromatic hyperedges. In this talk, we focus on a notion of linearly ordered k-colourings (LO_k-colourings) of hypergraphs. An LO_k-colouring of a hypergraph H assigns colours $\{1, \ldots, k\}$ to the vertices of H in a way that in each hyperedge the maximum colour appears exactly once. Observe that the problem of LO_2-colouring 3-uniform hypergraphs corresponds precisely to the classic NP-complete Positive-1-in-3-Sat problem (with vertices being interpreted as variables and hyperedges as clauses). This naturally leads to an approximation question: for $\ell \leq k \leq 2$, given an LO_k-colourable hypergraph, find its LO_ℓ-colouring.

We show that, if we restrict ourselves to 3-uniform hypergraphs, there exists an $\varepsilon > 0$ such that in polynomial time we can LO_ℓ-colour an LO_2-colourable graph for $\ell = (1 - \varepsilon) \log n$. The proof uses recent developments in the area of additive combinatorics, in particular the Polynomial Freiman-Ruzsa Theorem.

This is joint work with Benjamin Bedert, Tamio-Vesa Nakajima, and Standa Živný.

Bogdana Oliynyk
Silesian University of Technology

The Cayley graph of a quandle

In this talk, we present recent results on the structural properties of Cayley graphs associated with quandles, including explicit descriptions for several important classes of quandles. Originally introduced to model both conjugation in groups and the Reidemeister moves in knot theory, quandles have become one of the fundamental algebraic structures in low-dimensional topology. They one of a central tools in the algebraic study of knots and links, giving rise to a variety of computable invariants, including coloring invariants, cocycle invariants, and quandle homology. Beyond knot theory, quandles arise naturally in group theory, Hopf algebras, set-theoretic solutions of the Yang–Baxter equation, and the theory of symmetric spaces.

We show that the Cayley graph of a quandle, viewed as an ordinary graph, may be either symmetric or non-symmetric, and that the diameter of a connected component can be arbitrarily large. For an Alexander quandle $A_t(G)$ over a finite abelian group G, we prove that the connected components of the Cayley graph correspond to the cosets of the subgroup $\mathrm{im}(\mathrm{id}-t)$. For a generalized Alexander quandle $A_\phi(G)$ over a finite group G, we show that the Cayley graph is regular and that the in-degree and out-degree of each vertex are equal to the index of the subgroup of fixed points of the automorphism ϕ in G. When the defining automorphism is inner, we give an explicit description of the forward orbits and prove that the connected components correspond to the cosets of the subgroup generated by commutators with the defining element.

This is joint work with David Dolžan.

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Henri Ortmüller

University of Fribourg

Drawing strategies in Strong Ramsey games for 3-uniform hypergraphs

The *Strong Ramsey game* $\mathcal{R}(B, G)$ is a two player game with players P_1 and P_2, where B and G are k-uniform hypergraphs for some $k \geq 2$. G is always finite, while B may be infinite. P_1 and P_2 alternately color uncolored edges $e \in B$ in their respective color and P_1 begins. Whoever completes a monochromatic copy of G in their own color first, wins the game. If no one claims a monochromatic copy of G in a finite number of moves, the game is declared a draw.

In this talk, we present an infinite family of 3-uniform hypergraphs $\{G_t\}_{t\geq 3}$, such that P_2 has a drawing strategy in the Strong Ramsey game $\mathcal{R}(K^{(3)}_{\aleph_0}, G_t)$. This improves a result by David, Hartarsky, and Tiba (EuJC, 2020). Furthermore, we give a broad overview of the tools used in our proofs. Our results appear in: arXiv:2512.16722.

This is joint work with Nathan Bowler.

Juliana Palmen

AGH University

On 2-factorizations of $C(n, \{1, 3\})$

A *decomposition* of a graph G is a collection of its subgraphs $H_1, H_2, \ldots, H_t$ such that each edge of G belongs to exactly one H_i. We call it a *2-factorization* when every H_i is a 2-regular spanning subgraph of G.

Determining whether a graph admits a factorization into copies of a prescribed 2-factor is known as the Oberwolfach Problem (OP). While the classical problem concerns complete graphs, it can be formulated for any $2k$-regular graph and may also allow non-isomorphic factors. For example, the Hamilton–Waterloo Problem permits two distinct types of 2-factors.

The classical OP has been solved for several classes of 2-factors, including Hamiltonian cycles, bipartite 2-factors, and uniform 2-factors. Moreover, there exists an infinite family of complete graphs for which the OP has a solution for every 2-factor. These constructions are based on 2-factorizations of low-degree circulant graphs.

For a positive integer n and a connection set $S \subseteq \{1, \ldots, \lfloor(\frac{n}{2}\rfloor)\}$ a *circulant graph*, denoted by $C(n, S)$, is a graph $G = (V, E)$ such that $V = \mathbb{Z}_n$ and $E = \{\{u, v\} : \delta(u, v) \in S\}$ where $\delta(u, v) = \min\{\pm|u - v| \pmod{n}\}$.

In this talk, I will present results on 2-factorizations of the circulant graph $C(n, \{1, 3\})$ into both isomorphic and non-isomorphic factors, and discuss how factorizations and cycle decompositions of $C(n, \{1, 3\})$ can be used to construct solutions to other graph decomposition problems.

This is joint work with Mariusz Meszka.

Dömötör Pálvölgyi

Eötvös Loránd University

NCIS: Ramsey - a nearcircumsphere-Ramsey theorem for solvable transitive configurations

My talk will cover a new result in Euclidean Ramsey Theory, that every r-coloring of a sufficiently high-dimensional sphere of radius slightly larger than the circumradius of P contains a monochromatic congruent copy of P, assuming that P is a finite spherical set that admits a solvable group of isometries acting transitively on P. The proof builds on Kříž's group-theoretic argument and combines it with a topological method that may be of independent interest. In particular, I will show that if $n - 3k > C(r)$, then for every r-coloring of the directed edges of the Kneser graph there will be (A, B) and (B, C) that receive the same color, where A, B, C are pairwise disjoint k-sets from $\{1, \ldots, n\}$. I will attempt to present the main ideas without requiring any prior knowledge. Anyone interested in the details, or planning to leave early, can read the paper here.

Paweł Pękała
AGH University

On-line majority edge-colourings of graphs

A *majority edge-colouring* of a graph G is a colouring of the edges of G such that for each vertex v of G, at most half the edges incident with v have the same colour. This notion was introduced in 2023 by Bock, Kalinowski, Pardey, Pilśniak, Rautenbach and Woźniak who proved that every graph of minimum degree at least 2 has a majority 4-edge-colouring.

We investigate an *on-line* variant of this problem in which a graph is revealed edge-by-edge (or vertex-by-vertex) and an algorithm assigns colours to edges immediately after they are revealed. In particular, we show that the greedy colouring algorithm, which uses at most 5 colours, is optimal among on-line algorithms. In addition, we consider generalization of this problem to *on-line* $1/k$*-majority edge-colourings*.

Jan Petr

University of Passau

Distributive sublattices of Bruhat orders: the Harris-Kleitman inequality for finite Coxeter groups

Let W be a finite Coxeter group. The sorting orders introduced by Armstrong coarse the Bruhat order on W and turn W into a lattice. We classify those sorting orders that turn W into a distributive lattice and deduce that there is at least one such sorting order for every irreducible W with the sole exception given by $W = E_8$. As an application, we prove an analogue of the Harris–Kleitman inequality for finite Coxeter groups, which asserts that any two upward-closed subsets of W in the Bruhat order are positively correlated. This generalises both the original Harris–Kleitman inequality for hypercubes as well as the more recent analogue for symmetric groups proved by Johnson, Leader and Long.

This is joint work with Pavel Turek.

Marta Piecyk

CISPA Helmholtz Center for Information Security

Faster algorithms on subclasses of ordered graphs

An ordered graph consists of a graph G and a linear ordering $\prec$ of the vertex set $V(G)$. In my talk I will present results on the complexity of some classical graph problems restricted to certain classes of ordered graphs.

First, for a fixed constant t, I consider ordered graphs $(G, \prec)$ with vertices $v_1, \ldots, v_n$, where $v_i \prec v_j$ for $i < j$, such that for every $i \in [n-1]$, the number of edges with one endpoint in $\{v_1, \ldots, v_i\}$ and the other in $\{v_{i+1}, \ldots, v_n\}$ is bounded by t. Unordered graphs G that admit such a linear ordering $\prec$ are known as graphs of *cutwidth* at most t.

In this part of the talk, I will focus on the complexity of the Graph Homomorphism problem, which is a generalization of Coloring, parameterized by the cutwidth of the input graph, and its interesting connections to some other graph parameters.

Next, I consider ordered graphs that do not contain a fixed ordered graph H as an induced subgraph. I will present results on Maximum Weight Independent Set and List-k-Coloring on ordered H-free graphs, showing, respectively, complete and almost complete quasipolynomial/NP-hard complexity dichotomy.

Marcin Pilipczuk

University of Warsaw

A polynomial coreset for furthest neighbor in planar metrics

A furthest neighbor data structure on a metric space (V, dist) and a set $P \subseteq V$ answers the following query: given $v \in V$, output $p \in P$ maximizing $\mathrm{dist}(v, p)$; in the approximate version, it is allowed to report any $p \in P$ with $\mathrm{dist}(v, p) \geq (1 - \varepsilon) \max_{p' \in P} \mathrm{dist}(v, p')$ for an accuracy parameter $\varepsilon \in (0, 1)$. A particular type of approximate furthest neighbor data structure is an ε-coreset: a small subset $Q \subseteq P$ such that for every query $v \in V$ there is a feasible answer $p \in Q$.

Our main result is that in planar metrics there always exists an ε-coreset for furthest neighbors of size bounded polynomially in $(1/\varepsilon)$. This improves upon an exponential bound of Bourneuf and Pilipczuk [SODA'25] and resolves an open problem of de Berg and Theocharous [SoCG'24] for the case of polygons with holes.

On the technical side, we develop a connection between ε-coreset for furthest neighbors and an invariant of a metric space that we call an ε-*comatching index* — a sibling of ε-(semi-)ladder index, a.k.a, ε-scatter dimension, as defined by Abbasi et al [FOCS'23]. While the ε-(semi-)ladder index of planar metrics admits an *exponential* lower bound, we show that the ε-comatching index of planar metrics is *polynomial*, all in $1/\varepsilon$. The exponential separation between ε-(semi-)ladder and ε-comatching is rather surprising, and the proof is the main technical contribution of our work.

This is joint work with Kacper Kluk, Hung Le, Wojciech Nadara, Hector Tierno, and Vinayak.

Magdalena Prorok

AGH University

The Turán number for cycles of lengths congruent to 0 and 2 $\mod k$

We present minimum degree conditions which forces a graph to contain a cycle of length ℓ modulo k for fixed k and ℓ. Our outcomes improve the results of Gao, Huo, Liu and Ma [2]. Consequently, we determine the maximum number of edges in a graph that does not contain a cycle of length 0 modulo k for odd k and in a graph that does not contain a cycle of length 2 modulo k also for odd k.

This is joint work with Yandong Bai, Andrzej Grzesik, and Binlong Li.

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Gabriella Rácz

University of Debrecen

Interpretations of difference sequences of some combinatorial number sequences

The classical Bell numbers give the number of partitions of finite sets. Higher order difference sequences of the sequence of Bell numbers were investigated by several authors. In our talk, we collect some meanings of the numbers mentioned above including an interpretation published in Hungarian, therefore less known by the mathematical community, which we extend to study independent partitions of certain graphs.

We generalize some of these results to the difference sequences of the sequence of the so-called r-Bell numbers. We provide three interpretations: one related to enumerating set partitions having fixed largest singleton, another one in connection with the number of matchings in certain bipartite graphs, as well as the meaning about the number of independent partitions mentioned above.

If the blocks in the partition or the partition itself is ordered, then we obtain Bell–Lah and Fubini numbers, respectively. We extend our investigations for the difference sequences of the r-generalizations of these sequences.

This is joint work with Gábor Nyul.

Firdavs Rakhmonov

University of St Andrews

L^p AVERAGES OF THE DISCRETE FOURIER TRANSFORM AND APPLICATIONS

The discrete Fourier transform has proven to be an essential tool in many geometric and combinatorial problems in vector spaces over finite fields. In general, sets with good uniform bounds for the Fourier transform appear more 'random' and are easier to analyze. However, there is a trade-off: in many cases, obtaining good uniform bounds is not possible, even in situations where many points satisfy strong pointwise bounds. To address this limitation, a new approach has been proposed: one attempts to replace the need for *uniform* (L^∞) bounds with suitable bounds for the L^p *average* of the Fourier transform. In subsequent joint works, the authors applied this approach successfully to improve known results in the Fourier restriction problem in $\mathbb{F}_q^n$ and in the study of orthogonal projections in $\mathbb{F}_q^n$. In this talk, we discuss this general approach, provide several examples, and highlight some recent applications.

The talk is based on a series of papers with Jon Fraser.

Arsenii Sagdeev

Karlsruhe Institute of Technology

Monochromatic unit cycles in the plane

We show that for every integer $\ell \geq 2$, every 2-coloring of the plane contains a monochromatic set of 2ℓ points such that the unit segments between them form a 2ℓ-cycle $C_{2\ell}$. We also discuss odd cycles, higher dimensions, and larger number of colors.

Based on joint works with Nóra Almási, Maria Axenovich, Dingyuan Liu, and Kenneth Moore.

Nika Salia

Jagiellonian University

Triangles vs pentagons, and Berge cycles

How many triangles can a graph on n vertices have if it contains no cycle of length five? And how many hyperedges can a 3-uniform hypergraph have if it avoids a short Berge cycle? These two questions—one about counting triangles, the other about forbidding cycles in hypergraphs—turn out to be closely related, and both trace back to a construction of Bollobás and Győri obtained by doubling one color class of an extremal C_4-free bipartite graph.

A *Berge cycle of length* ℓ consists of distinct vertices $v_1, \dots, v_\ell$ and distinct hyperedges $e_1, \dots, e_\ell$ with $\{v_i, v_{i+1}\} \subseteq e_i$ for all i (indices modulo ℓ). Writing $\mathrm{ex}_3(n, \mathrm{BC}_\ell)$ for the maximum number of edges in an n-vertex 3-uniform hypergraph with no Berge cycle of length ℓ, and $\mathrm{ex}(n, K_3, C_5)$ for the maximum number of triangles in an n-vertex C_5-free graph, all of these extremal numbers have order $n^{3/2}$, and determining their leading constants is a recurring challenge.

In this talk I will survey this circle of problems and present recent progress. The main new result sharpens the leading constant for Berge-C_4-free hypergraphs: every n-vertex 3-uniform hypergraph with no Berge cycle of length four has at most

$$\frac{n^{3/2}}{2+\sqrt{2}} + O(n)$$

hyperedges, improving the previous best-known constant $1/\sqrt{10}$ to $1/(2+\sqrt{2})$. I will also discuss the companion problems of Berge 5-cycles in hypergraphs and of triangles in C_5-free graphs, the constructions giving the best known lower bounds, and the shared block-decomposition and path-counting techniques behind the upper bounds.

This talk is based on joint work with Casey Tompkins.

Harry Sapranidis Mantelos

The Hebrew University of Jerusalem

A cycle-based characterization of determinantal equivalence

Two bivariate functions $K, Q : \Lambda^2 \to \mathbb{F}$ are said to be determinantally equivalent if for any $n \in \mathbb{N}$ and $x_1, x_2, \ldots, x_n \in \Lambda$, the determinants of matrices $(K(x_i, x_j))_{1\leq i,j\leq n}$ and $(Q(x_i, x_j))_{1\leq i,j\leq n}$ agree. We study to what extent such functions K and Q must be related by two canonical transformations corresponding to diagonal similarity and transposition.

In the finite setting, this is closely related to the classical problem from [R. Loewy, *Principal minors and diagonal similarity of matrices*, Linear Algebra and its Applications 78, 1986, pp. 23–64] concerning principal minors and diagonal similarity of matrices. While previous approaches rely heavily on linear-algebraic techniques, we present a fundamentally different and entirely combinatorial method.

The key idea is to analyze permutations appearing in determinant expansions as cycles in a graph. In contrast to previous work in the nonvanishing setting [H. Sapranidis Mantelos, *Determinantally equivalent nonzero functions*, Discrete Mathematics 349(6):115021, 2026], the presence of zeros requires substantially new combinatorial arguments, as the identities used in that setting are no longer available. From the resulting cycle structure, we construct two auxiliary functions whose cocycle properties encode the possible equivalence mechanisms between K and Q. The main step is then to prove a structural dichotomy for 3-cycles, showing that all such cycles necessarily fall into a single class. This ultimately yields a precise description of determinantally equivalent functions under a natural structural assumption.

Based on my recent preprint arXiv:2604.03934.

Michał Seweryn

Jagiellonian University

Polynomial bounds for the Graph Minor Structure Theorem

The *Graph Minor Structure Theorem* describes approximately the structure of graphs which do not contain a fixed graph H as a minor; informally, such graphs can be obtained by gluing together smaller pieces, each of which consists of a graph embedded on a surface of small genus augmented with at most $f(t)$ "vortices" of "depth" at most $f(t)$, plus at most $f(t)$ "apices" — where, classically, f was not known to be better than a tower of exponentials. This theorem was proved by Robertson and Seymour as a tool for solving Wagner's Conjecture, and has since found numerous other applications in graph theory, algorithms, and combinatorial optimization.

In my talk, I will give an overview of the theory of graph minors, and discuss a recent proof of the Graph Minor Structure Theorem with polynomial bounds on $f(t)$.

This is joint work with Maximilian Gorsky and Sebastian Wiederrecht.

Dániel Gábor Simon

Alfréd Rényi Institute of Mathematics & Eötvös Loránd University

What are the possible values of the conductance of a graph?

The conductance h_G of a graph G (also known as the Cheeger constant) is a well studied parameter of a graph in mathematics and computer science. We study the unexplored problem of determining the set of possible values that h_G can achieve. We show that every rational in $[0, \frac{5}{9}]$ is achievable as a conductance and that only finitely many rationals in $(\frac{5}{9} + \varepsilon, 1]$ are achievable for all $\varepsilon > 0$. We further show that every graph with conductance close to $\frac{5}{9}$ must be close in edit distance to $K_{3,n}$, and that no graph has conductance in $(\frac{3}{4}, 1)$ which is best possible by considering $h_{K_3} = 1$ and $h_{K_5} = \frac{3}{4}$. This last result can be seen as a stability statement saying that $h_G < 1$ implies $h_G \leq \frac{3}{4}$.

This is joint work with Michal Bassan, Fiona Skerman, and Sam Spiro.

Tomasz Skalski

Wrocław University of Science and Technology

Level sets and maximum likelihood estimation for the Ising model

This talk concerns the existence of the maximum likelihood estimator (MLE) for a discrete exponential family on a finite set. Using the newly introduced notion of sets of uniqueness, we present new criterion for the existence of MLE. We apply this criterion in various discrete settings, in which it can be easily solved using the tools from random graph theory and discrete geometry. Most notably, we discuss the MLE existence for exponential models of random graphs and for linear spaces spanned by the products of Rademacher functions with special focus on the Ising model from statistical mechanics.

For the Ising model on a complete graph, we propose a full characterization among the level sets used in related combinatorial problems. As an application, we establish new bounds for the size of the smallest set of uniqueness for the products of Rademacher functions and discuss the asymptotics of the sample size for which the maximum likelihood estimator exists with high probability.

This is joint work with Tomasz Stroiński.

Bartosz Sobolewski

Jagiellonian University

Monochromatic arithmetic progressions in binary automatic words

Let $\mathbf{a} = (a_n)_{n\in\mathbb{N}}$ be an infinite binary word ($a_n \in \{0,1\}$), which we identify with a 2-coloring of nonnegative integers. The celebrated van der Waerden Theorem ensures that $\mathbf{a}$ contains arbitrarily long monochromatic arithmetic progressions (MAP for short). It is an interesting problem to locate these long MAPs and bound the lengths of MAPs in terms of their difference d, for various words $\mathbf{a}$ with complex structure. In recent years, several studies have addressed this type of questions for automatic and morphic words $\mathbf{a}$, such as Thue-Morse, Rudin-Shapiro, Fibonacci words, and more general families of related words.

In the present talk, we will give some new results on the Baum-Sweet word $\mathbf{b} = (b_n)_{n\in\mathbb{N}}$, defined by

$$b_n = \begin{cases} 0 & \text{if the binary expansion of } n \text{ contains a block of 0s of odd length,} \\ 1 & \text{otherwise.} \end{cases}$$

In contrast to the cases studied so far, $\mathbf{b}$ contains infinite MAPs, which we fully characterize. We therefore focus on MAPs starting at $n = 0$, which turn out to be finite, and give an upper bound on their length depending on the difference d. Moreover, we identify a sequence of differences d for which these MAPs are arbitrarily long.

This is joint work with Jan Zakrzewski.

Barbara Stołowska

AGH University

Total magic labellings

Given a finite Abelian group $(A, +)$, consider a graph G. Let labelling

$$f : V \cup E \to A \setminus \{0\}$$

induce labellings in G with general definition

$$S_x = \sum_{y: y \cap x \neq \emptyset} f(y).$$

The labelling is called $A-$*magic* labelling if all the sums S_x are equal. In this work we present results for three type of total magic labellings: edge, vertex and total.

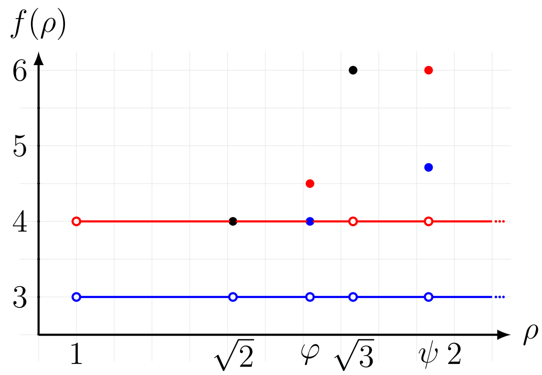
This is joint work with Mariusz Woźniak, Sylwia Cichacz-Przeniosło, and Agnieszka Gőrlich.

Cameron Strachan

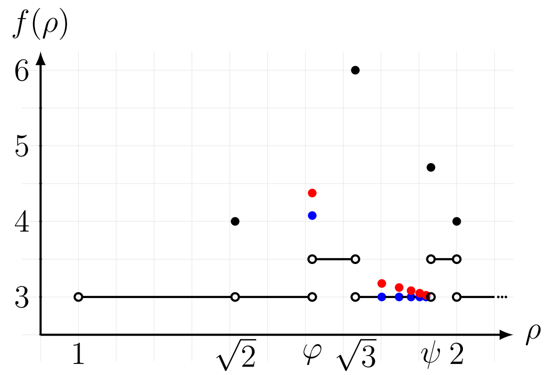
London School of Economics

The number of occurrences of the two smallest distances

For each value of $\rho > 1$, let $e(n, \rho)$ denote the maximum combined number of occurrences of the smallest and second smallest distances in a set of n points in the plane, given that the smallest distance is 1 and the second smallest distance is ρ. For all except six values of ρ, we determine the asymptotics of $e(n, \rho) = f(\rho)n - o(n)$. This extends the work of Vesztergombi and Csizmadia. For these exceptional values, we determine upper and lower bounds that are quite close, see Figure 1.



(a) Previous results



(b) New results

Figure 1: Lower bounds in blue, upper bounds in red, and known values in black

The hardest exceptional value is the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$, where we give the bounds $53/13 \leq f(\varphi) \leq 4.375$, with the lower bound disproving a conjecture of Csizmadia. It turns out that for all except finitely many values of ρ, $f(\rho) \in \{3, 7/2\}$. For $\rho = \varphi$, we find the upper bound by solving a large LP. For the other cases, we redraw the graph of the two smallest distances as a planar graph or a graph closely related to a planar graph.

This is joint work with Konrad Swanepoel.

Małgorzata Sulkowska

Wrocław University of Science and Technology

Community detection via star modularity

Modularity is a graph parameter introduced by Newman and Girvan in 2004. It measures the presence of community structure in graphs and is widely used in clustering applications. However, it exhibits several limitations, particularly the so-called resolution limit. This means that small communities are merged into larger ones since modularity is unable to detect structures below a certain scale. To address this issue, we introduce star modularity, a modification of the classical parameter. We analyze how star modularity behaves on sparse graphs in comparison with the traditional metric.

This is joint work with Lena Jasińska, Mikołaj Krupiński, and Zofia Kuriata.

Konstanty Junosza-Szaniawski
Warsaw University of Technology

Coloring mixed graphs

A mixed graph is a graph containing both undirected and directed edges. A proper coloring of a mixed graph is an assignment of positive integers to the vertices such that the endpoints of every undirected edge receive distinct colors, and for every directed edge (u, v), the color assigned to u is smaller than the color assigned to v.

Such colorings arise naturally in problems where one has to assign levels, labels, or time slots subject to both separation and precedence constraints. Examples include scheduling, timetabling, ranking, graph drawing, and related combinatorial optimization problems.

The corresponding decision problem is NP-complete, even for interval mixed graphs. In this talk, we introduce the problem of coloring mixed graphs, discuss its basic properties and applications, and present two exact algorithms for computing the minimum number of colors in a proper coloring. The first algorithm uses exponential space, whereas the second works in polynomial space. We also analyze their complexity and describe some connections with problems in extremal graph theory.

Raina Mary Thomas

Birla Institute of Technology and Science

ON THE COVERING ARRAY NUMBER OF 3-UNIFORM QUALITATIVE INDEPENDENCE HYPERGRAPHS

Covering arrays (CAs) are well-known combinatorial designs that generalize orthogonal arrays and facilitate efficient test suite generation in software testing. For a positive integer n and a set S of size g, a *covering array on a hypergraph* $H = (V, E)$ of size n and alphabet size g, denoted $CA(n, H, g)$, is an $n \times |V|$ array with entries from S, where columns correspond to the vertices in V, such that for every hyperedge $e \in E$ of size m, the sub-array indexed by the vertices of e contains all the ordered m-tuples from S^m at least once, as a row. The smallest n for which such an array exists is the *covering array number of* H, denoted $CAN(H, g)$. This parameter represents the minimal test suite size for practical applications. For a t-uniform hypergraph H and positive integers g and n, a $CA(n, H, g)$ exists if and only if there is a hypergraph homomorphism from H to t-$QI(n, g)$, a distinctive class of hypergraphs called *qualitative independence hypergraphs*. Consequently, for such a hypergraph H, $CAN(H, g) \leq CAN(t\text{-}QI(n, g), g)$. We develop a technique to determine the strong chromatic number using the existence of Latin squares; and the size of the largest 3-cliques of 3-$QI(n, 2)$, using Colbourn's technique of Derivation [1]. These results are then used to establish that $CAN(3\text{-}QI(n, 2), 2) = n$ for some values of n and obtain the covering array number of certain classes of hypergraphs.

This is joint work with Yasmeen Akhtar.

References

[1] C. J. Colbourn, G. Kèri, P. P. Rivas Soriano, and J.-C. Schlage-Puchta, *Covering and radius-covering arrays: constructions and classification*, Discrete Applied Mathematics 158(11), 2010, pp. 1158–1180.

Chefi Triki
University of Kent

An optimization framework for the coordination of the auction advertising and logistics operations

This study presents a combinatorial optimization framework for jointly coordinating freight transportation and digital advertising decisions under demand uncertainty. The proposed approach formulates the integrated planning problem as a multi-objective optimization model that simultaneously determines vehicle routing and sponsored search advertising strategies to maximize truck capacity utilization while controlling marketing expenditure. To explicitly capture the stochastic nature of customer demand, the model incorporates probabilistic constraints that regulate advertising decisions based on the likelihood of generating additional orders without exceeding available transportation capacity.

The resulting formulation integrates the capacitated vehicle routing problem with probabilistic demand stimulation into a unified combinatorial optimization model. The proposed framework is evaluated using data from a leading air-conditioner manufacturer and validated through live Google Search advertising campaigns. Computational results demonstrate significant improvements in fleet utilization, reducing unused truck capacity by more than 40% while increasing advertising return on investment by approximately 20%. These findings demonstrate that embedding probabilistic constraints within a combinatorial optimization framework provides an effective mechanism for synchronizing logistics and marketing decisions, leading to higher asset utilization and enhanced commercial performance.

This is joint work with Fathen Afandi.

Acknowledgement: Research reported in this abstract was supported by the Qatar Research Development and Innovation Council (ARG01-0430-230029). The content is solely the responsibility of the authors and does not necessarily represent the official views of the Qatar Research Development and Innovation Council.

Tom Trotter
Georgia Institute of Technology

Progress report on Boolean dimension and planarity

Since the 1980's, the following two conjectures regarding posets with planar cover graphs have challenged researchers: (1) Dushnik-Miller dimension is bounded in terms of standard example number; and (2) Boolean dimension is bounded. In 2022, Blake, Micek and Trotter settled both conjectures in the affirmative for posets with planar cover graphs and a unique minimal element. Joined by Hodor and Seweryn, we settled conjecture (1) in full generality. The 100 page manuscript is available via the arXiv, and the journal article is under review. In the intervening four years, there has been steady progress on conjecture (2), and now the end seems in sight.

This is joint work with Heather S. Blake, Jędrzej Hodor, Piotr Micek, and Michał Seweryn.

Mykhaylo Tyomkyn
Charles University

On plane rigidity matroids

Every connected cubic graph $H \neq K_4, K_{3,3}$ has an orientation without directed or alternating cycles. The current proof of this purely graph theoretic statement uses tropical geometry and matroid theory. The matroids in question are related to rigidity in $\mathbb{R}^2$ and naturally generalize the cycle matroid of K_n (forests/trees/cycles) to graphs with $2n-3$ edges. I will survey their remarkable properties and present some new ones.

Stepan Vakhrushev

Uppsala University

The threshold for a rainbow F-factors and a spread condition for colourings

We establish the threshold for a randomly coloured random graph to contain a rainbow copy of an F-factor (where F is any fixed 1-balanced graph), answering a question of Han and Yuan. Along the way we prove a theorem generalising recent work of Han and Yuan, where instead of rainbow colourings we consider a more general class of colourings satisfying a natural spread condition. The proof also involves several careful couplings including a coupling technique of McDiarmid (twice) and a coupling of Riordan which translates between copies of F in $G(n,p)$ and the r-uniform random hypergraph (where r is the number of vertices of F).

This is joint work with Annika Heckel.

Mamta Verma
Dr B. R. Ambedkar National Institute of Technology

Bounds for the spread of graph matrices

The spread of a graph matrix is an important spectral parameter that reflects structural properties of the underlying graph. In this work, we derive bounds for the spread of the adjacency, Laplacian, and signless Laplacian matrices. Our approach combines techniques from matrix analysis with inequalities for positive linear maps to obtain estimates in terms of fundamental graph parameters. The derived bounds improve many existing estimates and establish new relationships between spectral quantities and combinatorial properties of graphs. Examples are presented to illustrate the sharpness of the obtained bounds.

This is joint work with Ravinder Kumar.

Máté Vizer

Budapest University of Technology and Economics & Eötvös Loránd University

THE UNIFORM (p, q)-EXTREMAL PROBLEM FOR LARGE n

For integers $p \geq q \geq 2$, let $f_{p,q}(n, k)$ denote the maximum size of a family $\subseteq \binom{[n]}{k}$ with the following uniform Hadwiger–Debrunner type property: among any p members of , some q have non-empty common intersection. Write

$$p - 1 = t(q - 1) + r, \qquad 0 \leq r < q - 1.$$

A natural construction is obtained by taking all k-sets meeting a fixed t-set, and then adding r further k-sets disjoint from this centre. This gives

$$\binom{n}{k} - \binom{n-t}{k} + r$$

members, and was conjectured by Araujo-Pardo, Díaz-Patiño, Montejano and Oliveros to be optimal for fixed parameters and sufficiently large n.

We prove this conjecture for all fixed $p \geq q \geq 2$ and $k \geq 2$. More precisely, for all sufficiently large n,

$$f_{p,q}(n, k) = \binom{n}{k} - \binom{n-t}{k} + r,$$

and the only extremal families are exactly the canonical examples described above. The proof combines the large-star range of the Erdős Matching Conjecture with matching stability results of Frankl and Kupavskii. The main new ingredient is a multi-peeling argument which converts the local (p, q)-condition into a global matching bound on a large subfamily.

For $k \geq 3$, the same method also gives sharp stability below the Hilton–Milner scale: if a (p, q)-family has size within $M(n) = o(n^{k-1})$ of the optimum, then, for a unique t-set centre, it has at most r members outside the corresponding t-point star and misses at most $M(n)$ members from that star.

This is joint work with Barbara Anna Balázs.

Bartosz Walczak

Jagiellonian University

Recent progress on the Erdős-Hajnal problem on high-girth high-chromatic subgraphs

A classical result of Erdős asserts that for every g and k, there exists a graph with girth at least g (i.e., no cycles shorter than g) and chromatic number at least k. In the 1960s, Erdős and Hajnal conjectured a far-reaching strengthening of this result: for all g and k, there is a constant K such that every graph with chromatic number at least K contains a subgraph with girth at least g and chromatic number at least k. Let $f(g, k)$ denote the smallest such K (or infinity if no such K exists). Rödl proved the case $g = 4$ of the conjecture in 1976, with a tower-type upper bound on $f(4, k)$. Apart from Rödl's result, there was very little progress on the problem—until this year. In this talk, the following recent developments will be discussed.

1. Raphael Steiner proved an odd-girth variant of the conjecture, raised in 2018 by Mohar and Wu, in which only *odd* cycles of length at most g are forbidden in the requested subgraph. The resulting upper bound is an exponential tower of height $\frac{g-3}{2}$, which in particular yields a new, single exponential upper bound on $f(4, k)$. This is derived from recent superexponential lower bounds on the multicolor Ramsey numbers of the triangle and longer odd cycles using a remarkably simple argument.
2. For $g \geqslant 5$, Seth Pettie, Gábor Tardos, and the speaker proved a very large lower bound on $f(g, k)$, namely, the $\lceil \log_2 g \rceil$th function in the Ackermann hierarchy, which in the case $g = 5$ is the tower function. Here a key role is played by Burling's construction of triangle-free high-chromatic graphs, which exhibits various remarkable properties.
3. In September 17, Conjectures.io published an exposition of a formal proof that $f(5, 7) = \infty$ in Lean, which was submitted two days before by an anonymous user, thus disproving the Erdős-Hajnal conjecture.

The talk will conclude with some related open problems, including the fractional chromatic number variant of the conjecture, due to Bojan Mohar and Hehui Wu, for which the bounds in 1 and 2 above still apply.

Agnieszka Widz

Lodz University of Technology

Reviving the theory of infinite random graphs

The Rado graph, also known as the countable random graph, plays a central role in model theory, Fraïssé theory, and combinatorics. Up to isomorphism, it is the unique countable graph obtained with probability one when each pair of natural numbers is independently joined by an edge with probability one half. This uniqueness result, proved by Erdős and Rényi, prompted Erdős and Spencer to remark that "the theory of infinite random graphs is demolished by Erdős and Rényi's theorem." However, the classical construction relies on repeatedly tossing the same, possibly biased, coin. Our goal is to revive the study of infinite random graphs by allowing a different biased coin for each potential edge. This broader framework allows us to explore how far one can depart from the Rado graph while still obtaining a fixed isomorphism type with probability one. Particular attention is given to the question of which countable graphs can be drawn, that is, obtained almost surely by a suitable choice of edge probabilities.

This is joint work with Jarosław Swaczyna, Ziemowit Kostana, and Leonardo N. Coregliano.

Błażej Wróbel

Wrocław University of Science and Technology

Survivable networks design via random spanning trees

We consider the union of k random spanning trees of K_n, drawn uniformly at random. Since trees may share edges, multiple occurrences of an edge are replaced by a single edge to form a simple graph G. We study the random variable M counting surplus edge occurrences. We derive an exact formula for $\mathbb{E}[M]$ and show that $\mathbb{E}[M] < k(k-1)$ independently of n. This bound motivates a procedure for generating sparse simple k-edge-connected graphs.

This is joint work with Dominik Bojko.

Andrzej Żak
AGH University

Near-packings of graphs with bounded-degree overlap

A near-packing of a graph G is a permutation σ of its vertex set such that the common edges of G and its image $\sigma(G)$ form a graph belonging to a prescribed family $\mathcal{F}$. For a positive integer n and a family $\mathcal{F}$, let $m(n, \mathcal{F})$ denote the largest integer m such that every graph of order n and size at most m admits such a near-packing.

In this talk, we consider the families $\mathcal{D}_k$ of graphs with maximum degree at most k. The case $k = 0$ corresponds to classical graph packing and has been known for a long time, while the exact value of $m(n, \mathcal{D}_1)$ was determined only recently. We continue this line of research by studying the next natural case. In particular, we determine the asymptotic behaviour of $m(n, \mathcal{LF})$, where $\mathcal{LF}$ denotes the family of linear forests. As a consequence, we also obtain the asymptotic value of $m(n, \mathcal{D}_2)$.

Agata Żywot
Wrocław University of Science and Technology

Graphical complexity of direct products of permutation groups

The conditions under which the direct product of permutation groups can be realized as the automorphism group of an edge-colored graph are known; however, the number of colors used in such constructions is far from optimal. We refine these results by expressing the required number of colors in terms of those needed for the 2^*-closures of the component groups.

This is joint work with Mariusz Grech.