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The sum-free problem of Erdős

A set B is said to be sum-free if there are no x, y, z in B with $x + y = z$. One of the most elegant early applications of the probabilistic method in additive combinatorics is a classical argument of Erdős showing that any set of N integers contains a sum-free subset of size $N/3$, and this was later improved to $(N + 2)/3$ by Bourgain using an elaborate Fourier-analytic approach. A longstanding problem in additive combinatorics conjectures that one can always find a sum-free subset of size $N/3 + g(N)$, for some function g that tends to infinity. We discuss recent work confirming this conjecture.

This problem leads naturally to the study of estimating trigonometric sums. I will describe some new methods for doing this which combine Fourier analysis with ideas from additive combinatorics, and if time permits I will also discuss their applications to other problems, such as the lonely runner conjecture.