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Diameter thresholds of random Cayley graphs

Given a finite group G, the random Cayley graph model $\mathcal{G}(G,p)$ is defined as the probability space of all Cayley graphs of G where each element of G is included in the generating set independently at random with probability p. While $\mathcal{G}(G,p)$ shares several macroscopic similarities with the standard Erdős-Rényi model $G(n,p)$, such as threshold phenomena, its inherently regular and algebraic structure yields important differences.

In this talk, we will explore the threshold probabilities for the diameter of random graphs drawn from $\mathcal{G}(G,p)$. For a fixed target diameter $d \geqslant 2$, it is well-known that in $G(n,p)$, the threshold for transitioning to diameter at most d is tightly concentrated around $p \sim \sqrt[d]{\frac{2\log n}{n^{d-1}}}$. We will demonstrate that random Cayley graphs exhibit a similar phenomenon up to a constant factor.

Specifically, we establish that for any $\varepsilon > 0$, a graph in $\mathcal{G}(G,p)$, where G is a group of order N, has diameter at most d with high probability if

$$p \geqslant \sqrt[d]{(1+\varepsilon)d!\frac{\log N}{N^{d-1}}}$$

and diameter greater than d if

$$p \leqslant \sqrt[d]{\frac{1-\varepsilon}{2^d} \cdot \frac{\log N}{N^{d-1}}}$$

Crucially, we will highlight how the exact threshold constants in $\mathcal{G}(G,p)$, unlike in $G(n,p)$, depend fundamentally on the underlying family of groups being considered. We will conclude by discussing specific group families that demonstrate both our upper and lower bounds are essentially best possible.

This is joint work with Klas Markström, and Christina Savvidou.