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Majority C-coloring of graphs

Inspired by the majority colorings and C-colorings, we introduce and study the majority C-coloring of graphs [C. Bujtás, M. Dettlaff, H. Furmańczyk, A. Laskowska, *Majority C-coloring of graphs*, 2026]. In such a vertex coloring, every vertex shares its color with at least half of its neighbors.

Formally, let $G=(V,E)$ be an undirected graph and k a positive integer. A *majority C-coloring* ($\mathrm{C}_{\geqslant}$-coloring, for short) of G with k colors is a surjective function $\phi : V \to [k]$ such that every vertex $v \in V$ satisfies $\big|\{\, u \in N(v) : \phi(u) = \phi(v) \,\}\big| \geq \frac{1}{2}\deg_G(v)$. The largest integer k for which G admits a $\mathrm{C}_{\geqslant}$-coloring is called *majority C-chromatic number* ($\mathrm{C}_{\geqslant}$-chromatic number, for short) and it is denoted by $\overline{\chi}_{\geqslant}(G)$.

An upper bound on $\overline{\chi}_{\geqslant}(G)$ is proved in terms of the order, minimum, and maximum degree. Its sharpness is demonstrated by several results over different graph classes. In particular, $\overline{\chi}_{\geqslant}(P_n^k) = \overline{\chi}_{\geqslant}(C_n^k) = \lfloor n/(k+1) \rfloor$ is true for the k-th power of a path and a cycle if $n \geq k+2$. Further, $\overline{\chi}_{\geqslant}(G) = \frac{1}{3}(n-d)$ holds if G is a (claw, K_4)-free cubic graph and contains d diamonds. It is also shown that the majority C-chromatic number is not monotone under edge deletion. In fact, both the lower and upper bounds are sharp in the inequality chain $\overline{\chi}_{\geqslant}(G) - 2 \leq \overline{\chi}_{\geqslant}(G-e) \leq \overline{\chi}_{\geqslant}(G) + 1$. The minimum and maximum number of edges in an n-vertex graph G with $\overline{\chi}_{\geqslant}(G) = k$ are determined for every n and k. It is also pointed out that the classical chromatic number $\chi(G)$ and $\overline{\chi}_{\geqslant}(G)$ are incomparable, and the difference $\overline{\chi}_{\geqslant}(G) - \chi(G)$ can take any positive or negative integer. On the other hand, $\overline{\chi}_{\geqslant}(G) + \chi(G) \leq n+1$ holds for every graph G of order n. The decision problem of whether $\overline{\chi}_{\geqslant}(G) \geq k$ holds is NP-complete for every fixed $k \geq 2$. In contrast, some sufficient conditions for $\overline{\chi}_{\geqslant}(G) \geq 2$ are proved, and a linear-time algorithm is presented that determines $\overline{\chi}_{\geqslant}(T)$ if T is a tree.

This is joint work with Csilla Bujtás, Hanna Furmańczyk, and Aleksandra Laskowska.