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Finding near-optimal centerpoints via Tverberg points

An α-centerpoint of a set of n input points in $\mathbb{R}^d$, is a point such that any halfspace containing it, also contains an α-fraction of the input points. Helly's theorem implies the existence of a $1/(d+1)$-centerpoint for every finite set of points in $\mathbb{R}^d$. The question of *finding* such a centerpoint has algorithmic significance and a long history. Since the seminal work of Clarkson, Eppstein, Miller, Sturtivant and Teng [IJCGA, 1996], the fastest algorithmic options in this direction, have been either to find $\Omega(1/d^2)$-centerpoints in polynomial time in d [Clarkson et al. IJCGA, 1996, Har-Peled and Jones, SoCG 2019] or to find $\Omega(1/d)$-centerpoints, in time $O(d^d)$. Recently a remarkable result of Cherapanamjeri [*FOCS*, 2024] gave the first polynomial time randomized algorithm for computing *approximate* $\Omega(1/d)$-centerpoints, that is a point ε-close to the halfspaces that define any $\Omega(1/d)$-centerpoint. However the algorithm takes significantly more time than $O(d^{16})$, as it involves running an interior-point based subroutine for $\Omega\left(\left(\frac{d}{\varepsilon}\right)^{16}\right)$ steps.

We give a $\tilde{O}(d^5)$ time randomized algorithm for computing *exact* $\Omega\left((d \cdot \ln d \cdot \ln\ln d)^{-1}\right)$-centerpoints. Our output quality is within a $\frac{1}{\ln d \ln\ln d}$-factor of $(d+1)^{-1}$, which is the best guarantee possible. This improves significantly on the longstanding $d^{O(d)}$ running time of Clarkson et al. for obtaining $\omega(1/d^2)$-centerpoints.

Our main tool is a new randomized algorithm to compute points in the intersection of Tverberg partitions using a lift of Sarkaria [*Isr. J. Math.* 1991], together with sampling ideas from Soberón [*Combinatorica*, 2019].

As a simple corollary, we also use these ideas to obtain a new polynomial-time (in d) algorithm for the *Colorful Carathéodory* problem, which improves upon the $d^{O(\log d)}$ runtime algorithm of Mulzer and Stein [SoCG, 2015].

If time permits, I shall also discuss some further developments including an approximation algorithm for the Colorful Carathéodory problem, and its applications to the problem of finding Tverberg centers and eventually (approximate) $\Omega(1/d)$ centerpoints.