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On the partition function of a rectangle

A partition of a natural number n is a finite non-increasing sequence of positive integers whose sum is n. The total number of such sequences for n is counted by the partition function $p(n)$.

The story of integer partitions goes back to Euler, who, among other things, proved the celebrated pentagonal number theorem, and discovered the generating function for the partition function $p(n)$:

$$\sum_{n=0}^{\infty} p(n)q^n = \prod_{n=1}^{\infty} \frac{1}{1-q^n}.$$

There is an abundance of ways to generalize the partition function. We introduce its "geometric" extension, and investigate the number $p(m, n)$ of ways to partition a rectangle of size $m \times n$ into rectangular blocks with integer sides, where two partitions of the rectangle are considered the same if they consist of the same multiset of blocks (their geometric arrangement is neglected). We prove some basic properties of $p(m, n)$ and exhibit an analogue of the Hardy-Ramanujan formula in the case of $p(2, n)$. Moreover, we also describe the asymptotic behavior of the number of restricted partitions of a rectangle, where only blocks of some special sizes can be used as parts.

This is joint work with Robin Visser, and Maciej Zakarczemny.