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UNIVERSALLY SPECTRAL AND TRACE POSITIVE MATRIX WORDS

We discuss the following question: which words in noncommuting matrix variables and their formal transposes universally evaluate to matrices with nonnegative eigenvalues or nonnegative trace, regardless of the dimension or entries of the substituted matrices?

In this monomial setting, we provide a complete structural characterization analogous to Hermitian-square representations. This contrasts sharply with the broader noncommutative polynomial setting, where trace positivity is known not to be captured by Hermitian-square-type certificates. For individual words, however, we prove that both spectral and trace positivity force a rigid cyclic combinatorial square structure. This rigidity also extends naturally to the complex and infinite-dimensional settings.

As an application of our main theorems, we resolve questions originally posed by Lieb and Pierce, and later conjectured by Hillar and Johnson, motivated by quantum statistical mechanics. Our approach establishes a surprising connection to graph theory and yields a matrix analogue of the positive graph conjecture of Camarena, Csóka, Hubai, Lippner, and Lovász.

This is joint work with Fan Wei.