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LOCALLY IRREGULAR TOTAL COLORING AND STRONG LOCALLY IRREGULAR TOTAL COLORING OF GRAPHS

We say that a graph is *locally irregular* if adjacent vertices have different degrees. After a short introduction about the Local Irregularity Conjecture [1] and improved version of this conjecture stated by Sedlar and Škrekovski in [3] we present the concept of strongly locally irregular graphs introduced by Bensmail and Marcille in [2]. Then we introduce the Local Irregularity Conjecture for total graphs, where *total graph* is an ordered triple (V_0, V_1, E) , where V_0, V_1 are the sets of empty and full vertices, respectively, $V_0 \cap V_1 = \emptyset$, and the set of edges E is a subset of $\binom{V_0 \cup V_1}{2}$ ($E \cap (V_0 \cup V_1) = \emptyset$). By a *locally irregular* total graph $G = (V_0, V_1, E)$ we mean the total graph G where every pair of adjacent vertices has different *total degrees*. The *total degree* of a vertex v is the number of edges in E containing v plus one if $v \in V_1$ or plus zero otherwise. Then we present the set of results supporting this conjecture. In the end, we present the strong variant of this problem and the initial results on it.

This is joint work with Anna Flaszczyńska, Aleksandra Gorzkowska, Alfréd Onderko, and Mariusz Woźniak.

References

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- [2] J. Bensmail, C. Marcille, *Strongly locally irregular graphs and decompositions*, 2024.
- [3] J. Sedlar, R. Škrekovski, *Local irregularity conjecture vs. cacti*, Discrete Applied Mathematics 343, 2024, pp. 115–133.