Mateusz Kamyczura
AGH University

Conflict-free chromatic index

Let G be a graph with maximum degree Δ. An edge coloring of G is called closed conflict-free if the closed neighborhood of every edge contains a uniquely colored element. The open variant is defined analogously, excluding the edge itself from its neighborhood. The minimum number of colors required for such colorings are denoted as the closed and open conflict-free chromatic indices of G, respectively.

While the closed variant was previously known to be bounded by $3\log_2\Delta + 4$, the best known general upper bound for the open setting was $O(\log^{2+\epsilon}\Delta)$. In this work, we prove that for both variants, the conflict-free chromatic index is bounded above by $(1 + o(1))\log_2\Delta$ for all graphs with sufficiently large Δ. Since complete graphs require at least $(1-o(1))\log_2\Delta$ colors in both settings, our general upper bound is asymptotically tight in order and in the leading constant. Our proof combines the probabilistic method, including the Lovász Local Lemma and Chernoff bounds, with deterministic graph decomposition techniques.

This is joint work with Jakub Przybyło.