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## Saturation of oriented graphs

We consider a natural extension of the classical saturation problem to oriented graphs. We study the minimum number of edges in an oriented graph $H$ on $n$ vertices that avoids a fixed oriented graph $G$, but adding any edge to $H$ creates a copy of $G$.

Our main result shows that saturation functions of oriented graphs attain at least four distinct orders of magnitude. If $G$ is acyclic, then its saturation function is bounded if and only if $G$ contains an isolated edge; otherwise, it is linear. When the forbidden graph consists of a tournament together with an isolated edge, we identify nontrivial families whose saturation functions are bounded, as well as families whose saturation functions are linear. The remaining two orders of magnitude are $\Theta(n\sqrt{\log n})$ and $\Theta(n \log n)$. The corresponding lower bounds are obtained by adapting a recently developed technique for the saturation problem in edge-ordered graphs, while the matching upper bounds are constructive. The first family consists of a specific class of oriented graphs, while the second applies to a broader family of oriented graphs in which every edge is contained in an oriented triangle, subject to mild additional assumptions.

Finally, we also study the semisaturation problem, in which we require only that adding any missing edge to $H$ creates a copy of $G$. For this variant, we completely characterize the possible orders of magnitude: the semisaturation function is either bounded, linear, or $\Theta(n \log n)$.

This is joint work with Vladimir Bošković.