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SOLUTION OF UNIFORM TURÁN'S TETRAHEDRON PROBLEM

Abstract

Turán's Tetrahedron Problem asks to determine the Turán density of the complete hypergraph $K_4^{(3)}$ (tetrahedron). This problem, posed by Turán in 1941, is one of the most famous problems in extremal combinatorics and its solution would attract \$500 prize from Erdős. In the 1980s, Erdős and Sós asked to determine Turán densities of $K_4^{(3)-}$ (broken tetrahedron) and $K_4^{(3)}$ (tetrahedron) when edges are constrained to be uniformly distributed in the host hypergraph. The presumably easier case of the broken tetrahedron was solved by Glebov, Král' and Volec [Israel J. Math. 211 (2016), 349–366] and Reiher, Rödl and Schacht [J. Eur. Math. Soc. 20 (2018), 1139–1159]. We solve the tetrahedron case by proving that the uniform Turán density of $K_4^{(3)}$ is equal to $1/2$; this confirms that Rödl's lower bound construction from 1986 is optimal.

This is joint work with Daniel Král', Ander Lamaison, Hong Liu, Xichao Shu, and Zhuo Wu.