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LISN

Graph reconstruction via subgraph queries

Bastide, Cook, Erickson, Groenland, Kreveld, Mannens, and Vermeulen (WG 2023) introduced the problem of reconstructing graphs from queries on triples of vertices, where each query asks whether the induced subgraph is connected or disconnected. This framework was later generalized by Galliot, La, Maistre, Petiteau, and Watel (EUROCOMB 2025), who considered partitions of the set of all isomorphism classes of graphs: the answer to a query is the part containing the isomorphism class of the induced subgraph. For instance, connectedness queries correspond to the bipartition in which one part contains all connected graphs and the other all disconnected graphs, up to isomorphism. This generalized setting also captures the classical Graph Reconstruction Conjecture of Kelly (1957) and Ulam (1960), which asserts that every graph on at least three vertices is uniquely determined by its induced subgraphs obtained by deleting one vertex.

Kluk, La, and Piecyk (WG 2024) showed that reconstructing graphs from variants of connectedness queries is hard, and that even for restricted graph classes, graphs with identical answers to all queries need not be unique. Motivated by this, Galliot et al. asked which partitions, for general tuples of vertices, guarantee unique reconstruction for all sufficiently large graphs.

For every integer $k \geq 2$, we give necessary and sufficient conditions on a partition Π of the isomorphism classes of graphs of order k such that every graph on at least $R(R(k))$ vertices is uniquely reconstructible from its answers to all k-vertex queries with respect to Π, where R denotes the diagonal Ramsey number. Our proof is constructive and yields a reconstruction algorithm using all queries, with running time $O(n^{Ck})$ for some fixed constant C. We also prove a lower bound of order $R(k)$: under only the necessary conditions on Π, there exist many graphs on fewer than $R(k)$ vertices that are not uniquely determined by their query answers.

This is joint work with Johanne Cohen, and Bianca Oancea.