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COARSE PROPERTIES OF VERTEX-MINOR CLOSED CLASSES OF GRAPHS

A graph H is a *vertex-minor* of G if it can be obtained from G via a sequence of local complementations and vertex deletions. Vertex-minors exhibit numerous structural and algorithmic properties analogous to those of traditional graph minors. The class of circle graphs is vertex-minor closed and serves as a fundamental building block in Geelen’s Structure Conjecture, the vertex-minor analog to the Robertson–Seymour Graph Minors Structure Theorem. This conjecture describes graphs in a proper vertex-minor closed class in terms of circle graphs, subjected to two operations: bounded perturbations and a more intricate construction. Although Geelen’s conjecture does not hold in full generality, these objects and operations remain of independent interest from the vertex-minor perspective.

Parallel to these developments, the emerging field of Coarse Graph Theory has begun bridging coarse geometry and structural graph theory, as explored in the works of Georgakopoulos and Papasoglu, the “Asymptotic Structure” series by Nguyen, Scott, and Seymour, and many other works. Central to this field is the study of *coarse tree decompositions*, i.e. tree decompositions whose bags are coverable by a fixed number of balls of bounded radius.

As part of an effort to bridge these two areas of research, we present bounds on coarse properties for graph classes related to Geelen’s Structure Conjecture. In particular, we provide bounds on the coarse treewidth of circle graphs and of bounded perturbations of graphs in proper vertex-minor closed classes.

These are results of my Master’s thesis project under supervision of Michał Pilipczuk.