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TURÁN PROBLEMS FOR SIMPLICIAL COMPLEXES

An abstract simplicial complex $\mathbf{F}$ is a non-uniform hypergraph without isolated vertices whose edge set is closed under taking subsets. The extremal number $\text{ex}(n, \mathbf{F})$ is defined as the maximum number of edges in an $\mathbf{F}$ -free simplicial complex on n vertices. Although Turán-type problems for simplicial complexes have long appeared in extremal set theory, a systematic study of $\text{ex}(n, \mathbf{F})$ was initiated only recently by Conlon, Piga, and Schülke. In contrast to uniform hypergraphs, even the order of magnitude of $\text{ex}(n, \mathbf{F})$ remains unknown for most simplicial complexes.

In this talk, we present a general framework for estimating $\text{ex}(n, \mathbf{F})$ via generalised Turán numbers of associated hypergraphs. Using this approach, we determine the asymptotic behaviour, and in some cases the exact value, of $\text{ex}(n, \mathbf{F})$ for large classes of simplicial complexes, extending a result of Conlon, Piga, and Schülke. Several open problems concerning the potential behaviour of $\text{ex}(n, \mathbf{F})$ are formulated.

This is joint work with Maria Axenovich, Dániel Gerbner, and Balázs Patkós.