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ASYMPTOTIC DIMENSION OF GRAPHS EXCLUDING
AN INDUCED OR FAT MINOR

Asymptotic dimension is a large-scale invariant of metric spaces that was introduced by Gromov (1993). A major open problem in coarse graph theory asks whether, for every fixed graph H , the class of graphs excluding H as a fat minor has bounded asymptotic dimension. A weaker conjecture asserts the same conclusion for graphs excluding H as an induced minor.

We substantially expand the range of graphs H for which these conjectures are known to hold. Specifically, we prove bounded asymptotic dimension for the following classes of graphs: graphs excluding a fixed planar graph as a fat minor; and graphs excluding a fixed string graph as an induced minor. All of our results also hold for the stronger notion of Assouad–Nagata dimension, a refinement of asymptotic dimension.

These results have a number of applications, two of which we mention here. First they imply that an induced minor-closed class is χ -bounded if and only if it does not contain some Burling graph. Second they imply that, in graph classes excluding a fixed planar graph as an induced minor, treewidth is polynomial in maximum degree.

This is joint work with Robert Hickingbotham, Freddie Illingworth, and Bartosz Walczak.