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## Almost tight bounds for induced subdivisions

Subdivisions of complete graphs play a central role in combinatorics, having deep connections to structural, extremal, and topological aspects of graph theory. A celebrated conjecture of Mader, proved independently by Bollobás and Thomason and by Komlós and Szemerédi, states that every graph of average degree of order $h^2$ contains a subdivision of $K_h$.

We consider the induced variant of this problem. A theorem of Kühn and Osthus implies that, for every fixed graph $H$ and every $s \geq 1$, graphs of sufficiently large average degree contain either a copy of $K_{s,s}$ or an induced subdivision of $H$. However, even for $H = K_h$, the best previous quantitative bounds were far from optimal.

We prove nearly tight bounds for forcing induced subdivisions of $K_h$. We show that every $K_{s,t}$-free graph of average degree $\Omega_{s,t}(h^{2(s-1)} \log^{7(s-1)} h)$ contains an induced subdivision of $K_h$, and that every $C_{2k}$-free graph with $k \geq 3$ and average degree $\Omega_k(h \log^5 h)$ contains an induced subdivision of $K_h$. These bounds substantially improve the previously known results and are nearly optimal in both settings. They also hold if $K_h$ is replaced by any other graph on $h$ vertices.

This is joint work with Zach Hunter, Aleksa Milojević, and Benny Sudakov.