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Local computation of parity in cellular automata

Cellular automata (CAs) provide a simple yet powerful model of distributed computation in which each cell updates its state according to a fixed local rule. Despite relying exclusively on local interactions, cellular automata are capable of performing nontrivial global computations, making them an attractive framework for studying the relationship between locality and computational complexity.

Among the best-known benchmark problems in this area is the parity problem. Given a binary cyclic configuration of odd length, the objective is to design a cellular automaton that converges to the homogeneous configuration of all 0s when the initial number of 1s is even, and to the homogeneous configuration of all 1s otherwise. Since each cell has access only to a finite neighborhood, solving the problem requires the automaton to compute a global property without centralized control.

For many years it was unknown whether such a cellular automaton exists. The first proposed solution, known as the BFO rule, marked a significant breakthrough, although subsequent work revealed inaccuracies in its original formulation. A corrected version, accompanied by a complete proof of correctness, eventually established that the parity problem is indeed solvable.

The existence of a solution naturally raises a number of further questions. In particular, it remains unclear whether the parity problem can be solved by a simpler cellular automaton and what the minimum neighborhood size required for such a solution is. Recent advances provide new tools for investigating these questions.

This is joint work with Barbara Wolnik.