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HYPERGRAPH COLOURINGS AND STRONG SPARSIFICATION

The classic notion of (vertex) graph colouring can be generalized to hypergraphs in several interesting ways; the most popular one avoids monochromatic hyperedges. In this talk, we focus on a notion of linearly ordered k -colourings (LO_k -colourings) of hypergraphs. An LO_k -colouring of a hypergraph H assigns colours $\{1, \dots, k\}$ to the vertices of H in a way that in each hyperedge the maximum colour appears exactly once. Observe that the problem of LO_2 -colouring 3-uniform hypergraphs corresponds precisely to the classic NP-complete POSITIVE-1-IN-3-SAT problem (with vertices being interpreted as variables and hyperedges as clauses). This naturally leads to an approximation question: for $\ell \leq k \leq 2$, given an LO_k -colourable hypergraph, find its LO_ℓ -colouring.

We show that, if we restrict ourselves to 3-uniform hypergraphs, there exists an $\varepsilon > 0$ such that in polynomial time we can LO_ℓ -colour an LO_2 -colourable graph for $\ell = (1 - \varepsilon) \log n$. The proof uses recent developments in the area of additive combinatorics, in particular the Polynomial Freiman-Ruzsa Theorem.

This is joint work with Benjamin Bedert, Tamio-Vesa Nakajima, and Standa Žitný.