Bogdana Oliynyk

Silesian University of Technology

The Cayley graph of a quandle

In this talk, we present recent results on the structural properties of Cayley graphs associated with quandles, including explicit descriptions for several important classes of quandles. Originally introduced to model both conjugation in groups and the Reidemeister moves in knot theory, quandles have become one of the fundamental algebraic structures in low-dimensional topology. They one of a central tools in the algebraic study of knots and links, giving rise to a variety of computable invariants, including coloring invariants, cocycle invariants, and quandle homology. Beyond knot theory, quandles arise naturally in group theory, Hopf algebras, set-theoretic solutions of the Yang–Baxter equation, and the theory of symmetric spaces.

We show that the Cayley graph of a quandle, viewed as an ordinary graph, may be either symmetric or non-symmetric, and that the diameter of a connected component can be arbitrarily large. For an Alexander quandle $A_t(G)$ over a finite abelian group G, we prove that the connected components of the Cayley graph correspond to the cosets of the subgroup $\mathrm{im}(\mathrm{id}-t)$. For a generalized Alexander quandle $A_\phi(G)$ over a finite group G, we show that the Cayley graph is regular and that the in-degree and out-degree of each vertex are equal to the index of the subgroup of fixed points of the automorphism ϕ in G. When the defining automorphism is inner, we give an explicit description of the forward orbits and prove that the connected components correspond to the cosets of the subgroup generated by commutators with the defining element.

This is joint work with David Dolžan.

References

[1] D. Dolžan, B. Oliynyk, *The Cayley graph of a quandle*, arXiv:2604.17011.