Juliana Palmen

AGH University

On 2-factorizations of $C(n,\{1,3\})$

A *decomposition* of a graph G is a collection of its subgraphs $H_1, H_2, \ldots, H_t$ such that each edge of G belongs to exactly one H_i. We call it a *2-factorization* when every H_i is a 2-regular spanning subgraph of G.

Determining whether a graph admits a factorization into copies of a prescribed 2-factor is known as the Oberwolfach Problem (OP). While the classical problem concerns complete graphs, it can be formulated for any $2k$-regular graph and may also allow non-isomorphic factors. For example, the Hamilton–Waterloo Problem permits two distinct types of 2-factors.

The classical OP has been solved for several classes of 2-factors, including Hamiltonian cycles, bipartite 2-factors, and uniform 2-factors. Moreover, there exists an infinite family of complete graphs for which the OP has a solution for every 2-factor. These constructions are based on 2-factorizations of low-degree circulant graphs.

For a positive integer n and a connection set $S \subseteq \{1, \ldots, \lfloor(\frac{n}{2}\rfloor)\}$ a *circulant graph*, denoted by $C(n, S)$, is a graph $G = (V, E)$ such that $V = \mathbb{Z}_n$ and $E = \{\{u, v\} : \delta(u, v) \in S\}$ where $\delta(u, v) = \min\{\pm|u - v| \pmod{n}\}$.

In this talk, I will present results on 2-factorizations of the circulant graph $C(n,\{1,3\})$ into both isomorphic and non-isomorphic factors, and discuss how factorizations and cycle decompositions of $C(n,\{1,3\})$ can be used to construct solutions to other graph decomposition problems.

This is joint work with Mariusz Meszka.