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A polynomial coreset for furthest neighbor in planar metrics

A furthest neighbor data structure on a metric space (V, dist) and a set $P \subseteq V$ answers the following query: given $v \in V$, output $p \in P$ maximizing $\mathrm{dist}(v, p)$; in the approximate version, it is allowed to report any $p \in P$ with $\mathrm{dist}(v, p) \geq (1-\varepsilon)\max_{p' \in P} \mathrm{dist}(v, p')$ for an accuracy parameter $\varepsilon \in (0, 1)$. A particular type of approximate furthest neighbor data structure is an ε-coreset: a small subset $Q \subseteq P$ such that for every query $v \in V$ there is a feasible answer $p \in Q$.

Our main result is that in planar metrics there always exists an ε-coreset for furthest neighbors of size bounded polynomially in $(1/\varepsilon)$. This improves upon an exponential bound of Bourneuf and Pilipczuk [SODA'25] and resolves an open problem of de Berg and Theocharous [SoCG'24] for the case of polygons with holes.

On the technical side, we develop a connection between ε-coreset for furthest neighbors and an invariant of a metric space that we call an ε-*comatching index* — a sibling of ε-(semi-)ladder index, a.k.a, ε-scatter dimension, as defined by Abbasi et al [FOCS'23]. While the ε-(semi-)ladder index of planar metrics admits an *exponential* lower bound, we show that the ε-comatching index of planar metrics is *polynomial*, all in $1/\varepsilon$. The exponential separation between ε-(semi-)ladder and ε-comatching is rather surprising, and the proof is the main technical contribution of our work.

This is joint work with Kacper Kluk, Hung Le, Wojciech Nadara, Hector Tierno, and Vinayak.