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Monochromatic arithmetic progressions in binary automatic words

Let $\mathbf{a} = (a_n)_{n\in\mathbb{N}}$ be an infinite binary word ($a_n \in \{0,1\}$), which we identify with a 2-coloring of nonnegative integers. The celebrated van der Waerden Theorem ensures that $\mathbf{a}$ contains arbitrarily long monochromatic arithmetic progressions (MAP for short). It is an interesting problem to locate these long MAPs and bound the lengths of MAPs in terms of their difference d, for various words $\mathbf{a}$ with complex structure. In recent years, several studies have addressed this type of questions for automatic and morphic words $\mathbf{a}$, such as Thue-Morse, Rudin-Shapiro, Fibonacci words, and more general families of related words.

In the present talk, we will give some new results on the Baum-Sweet word $\mathbf{b} = (b_n)_{n\in\mathbb{N}}$, defined by

$$b_n = \begin{cases} 0 & \text{if the binary expansion of } n \text{ contains a block of 0s of odd length,} \\ 1 & \text{otherwise.} \end{cases}$$

In contrast to the cases studied so far, $\mathbf{b}$ contains infinite MAPs, which we fully characterize. We therefore focus on MAPs starting at $n = 0$, which turn out to be finite, and give an upper bound on their length depending on the difference d. Moreover, we identify a sequence of differences d for which these MAPs are arbitrarily long.

This is joint work with Jan Zakrzewski.