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THE UNIFORM (p, q) -EXTREMAL PROBLEM FOR LARGE n

For integers $p \geq q \geq 2$, let $f_{p,q}(n, k)$ denote the maximum size of a family $\subseteq \binom{[n]}{k}$ with the following uniform Hadwiger–Debrunner type property: among any p members of $\mathcal{F}$, some q have non-empty common intersection. Write

$$p - 1 = t(q - 1) + r, \quad 0 \leq r < q - 1.$$

A natural construction is obtained by taking all k -sets meeting a fixed t -set, and then adding r further k -sets disjoint from this centre. This gives

$$\binom{n}{k} - \binom{n-t}{k} + r$$

members, and was conjectured by Araujo-Pardo, Díaz-Patiño, Montejano and Oliveros to be optimal for fixed parameters and sufficiently large n .

We prove this conjecture for all fixed $p \geq q \geq 2$ and $k \geq 2$. More precisely, for all sufficiently large n ,

$$f_{p,q}(n, k) = \binom{n}{k} - \binom{n-t}{k} + r,$$

and the only extremal families are exactly the canonical examples described above. The proof combines the large-star range of the Erdős Matching Conjecture with matching stability results of Frankl and Kupavskii. The main new ingredient is a multi-peeling argument which converts the local (p, q) -condition into a global matching bound on a large subfamily.

For $k \geq 3$, the same method also gives sharp stability below the Hilton–Milner scale: if a (p, q) -family has size within $M(n) = o(n^{k-1})$ of the optimum, then, for a unique t -set centre, it has at most r members outside the corresponding t -point star and misses at most $M(n)$ members from that star.

This is joint work with Barbara Anna Balázs.